Towards a Strategic Satisfactory Sensing for QoS Self-provisioning in Cognitive Radio Networks

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***Abstract*—In cognitive radio networks, secondary users (SUs) face two conflicting objectives. Each SU seeks to minimize the sensing duration while maximizing the detection probability of primary users (PU) to avoid interfering with their transmissions. Both objectives have a substantial effect on energy efficiency. This paper investigates a noncooperative setting for selecting the sensing duration when multiple SUs operate in the same network. Here, each SU has a certain throughput requirement. The interaction among SUs is captured via a satisfaction strategic game with explicitly stated throughput demands. We prove that depending on the throughput requirements, either zero, one or two Satisfaction Equilibria (SE) exist. We then provide a fully distributed learning algorithm (SELA) to discover them. Extensive simulation results show the validity of the proposed SELA and illustrate the relationship between the throughput demand and the sensing duration.**

Index Terms: Cognitive Radio Networks, Strategic Sensing, Satisfaction Equilibrium, Learning Algorithm, QoS.

I. Introduction

In Spectrum-adaptive cognitive radio networks (CRNs), secondary users (SUs) are granted access to the spectrum holes when the primary users (PUs) are not communicating. To avoid interfering with PUs, channel sensing is performed by SUs before attempting any transmission. Sensing techniques fall in one of the following three categories [1]: energy detection, matched filtering and feature detection. Cooperative sensing among multiple SUs may also be used to improve sensing accuracy and cope with channel fading and interference.

The authors of [2] tackled the problem of uncertain capacity in CRNs. They provided a QoS model to investigate the performance of SUs in terms of approximated delay violation probability and mean time delay. To address sensing errors, they proposed a framework for collaborative sensing between SUs. In [3] a spectrum sensing algorithm for CR with QoS support is proposed. To optimize the sensing duration, parallel sequential probability ratio tests (SPRTs) were used. The proposed approach is benchmarked against fixed-sample size (FSS) detectors and its performance advantage was demonstrated. In [4] joint optimization of the spectrum sensing and data transmission with amplify-and-forward relays are considered. This approach significantly improves SU's throughput in comparison with other mechanisms. The work in [5] optimizes the total throughput of SUs as a trade-off between an access parameter p and sensing design. The objective is to optimize the channel assignment to SUs and the associated sensing times. One interesting feature of this work is its cross-layer nature. Indeed, both medium access layer (p-persistent CSMA-based) and physical layer (semi-distributed cooperative spectrum sensing) are considered in the optimization. In [6], the authors cast the spectrum sensing window optimization into a convex optimization problem. The sensing duration for SUs operating on Ultra Wide Band (UWB) is optimized subject to constraints on detection and false-alarm probabilities. A two-step game for joint sensing and opportunistic access is presented in [7]. The authors provide a characterization of the Nash equilibria and a combined distributed learning algorithm to help SUs determine their optimal payoffs and strategies.

In this paper, we propose a satisfactory sensing approach based on energy detection. We extend the work of [8] by providing a distributed mechanism for SUs to discover their optimal sensing durations based on their satisfaction levels. These satisfaction levels are expressed in terms of throughput requirements.

Our contribution is threefold. First, we formulate the problem of choosing the duration of the sensing period to meet a given SU's throughput requirement as a noncooperative satisfactory sensing game. Second, we prove that the proposed game has either zero, one, or two satisfaction equilibria (SEs). Finally, we provide a distributed learning algorithms for the SUs to obtain their optimal sensing durations along with a selection approach of satisfaction equilibrium based on the value of the miss-detection probability.

The remainder of this paper is organized as follows. Section II presents the channel sensing model and lists the main assumptions. The satisfactory sensing game is presented and its Nash/Satisfaction equilibria existence and enumeration stated in Section III, which is followed in Section IV by the proposed fully distributed Satisfactory Equilibria Learning Algorithm

(SELA). Numerical results are presented and discussed in Section V. Section VI provides some conclusions and directions for future work.

In the rest of the paper, we will use the bold notation for vectors and normal notation for scalars.

II. Channel Sensing Model

We consider a set $\mathcal{N}$ of SUs, with cardinality N, competing to access a licensed spectrum when it is unused by PUs. SUs are symmetric in the sense that they will attempt to transmit if the channel is sensed to be free with the same probability q. However, SUs have different throughput requirements $\{r_i\}_{i\in\mathcal{N}}$. Each SU controls its sensing duration before attempting a transmission. The frame duration of SUs T divides into the sensing and transmission periods.

Throughout this paper, we work under the following assumptions:

- Energy-based spectrum sensing: PUs activity is determined by measuring the received signal strength. If the received signal power exceeds a predefined threshold ϵ, the channel is declared busy. Otherwise, it is declared idle and SUs may attempt to transmit.
- Due to imperfect sensing, SUs may either declare idle a busy channel (miss-detection) or busy an idle one (false-alarm).
- Frame-based time-synchronized approach: all SUs transmitters start simultaneously at the beginning of each frame. Each frame consists of a sensing period, followed by data transmission (if any) for the rest of the frame.
- Homogeneous spectrum environment, i.e, the state of the PU is the same at both ends of the SU link.
- SUs adopt a slotted aloha-like protocol as their medium access control protocol.

During primary users' activities, each SU samples the received signal at sampling frequency f_s. Without loss of generality, we assume that all SUs use the same sampling frequency. The discrete received signal at the SU i can be represented as

$$y_i(t) = \begin{cases} h_i.s(t) + n(t) & : \text{Hypothesis } \mathcal{H}_1(\text{Busy}) \\ n(t) & : \text{Hypothesis } \mathcal{H}_0(\text{Idle}) \end{cases} \quad (1)$$

where $s(t)$ is the transmitted signal, h_i is the channel gain experienced by SU i and $n(t)$ is a circularly symmetric complex Gaussian noise with mean 0 and variance $E[|n(t)|^2] = \sigma^2$. The channel state is considered as the binary hypothesis test $\mathcal{H}_0$ and $\mathcal{H}_1$.

Let τ be the sensing duration and M the number of samples. Thus, we have $M = \tau f_s$. It follows that the average energy detected by SU i is

$$T_i(M) = \frac{1}{M}\sum_{i=1}^{M} |y_i(t)|^2. \quad (2)$$

A key performance metric for CRNs is the false-alarm probability. This metric corresponds to the case where the channel is declared busy when it is idle in reality. When energy detection is adopted, the false-alarm probability is given by:

$$P_{f_a}(\tau_i) = \frac{1}{2}\text{erfc}(A\sqrt{\tau_i}),\ A \triangleq \frac{(\epsilon - \sigma^2)\sqrt{f_s}}{\sigma^2}. \quad (3)$$

where erfc(.) is the complementary error function.

III. Satisfactory sensing game

The sensing duration setting involves multiple SUs who choose independently the strategy maximizing their throuhput. The SU i picks a value within $[0, T]$ for its sensing duration τ_i. From a single SU perspective, there is a trade-off between the false-alarm probability and the transmission phase duration. On one hand, as the sensing duration increases, the false-alarm probability P_{f_a} decreases. On the other hand, the transmission phase duration and the achievable throughput decrease. Besides, as SUs contend for channel access, the sensing duration of each SU will impact the overall interference probability and consequently other SUs achievable throughput. This setup can be naturally addressed using noncooperative game theory.

A. Mathematical Model

We model the distributed sensing based on energy detection for SUs as noncooperative strategic satisfactory sensing game:

$$\mathcal{G} = \{\mathcal{N}, \{\mathcal{A}_i\}_{i\in\mathcal{N}}, \{\mathcal{U}_i\}_{i\in\mathcal{N}}, \mathcal{R}\} \quad (4)$$

where:

- $\mathcal{A}_i = [0, T]$ Every SU will choose a strategy $\tau_i \in [0, T]$ for its sensing duration. The strategy profile of the game $\mathcal{G}$ is $\boldsymbol{\tau} = (\tau_i, \boldsymbol{\tau_{-i}})$, which denotes the vector of the strategies chosen by the SU i and the other SUs are denoted by $-i$ (i.e, $-i \triangleq \mathcal{N} \setminus i$).
- $\mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}})$ represents the payoff for SU i when selecting strategy τ_i, when the other SUs choose the sensing durations vector $\boldsymbol{\tau_{-i}}$. Specifically, we take $\mathcal{U}_i$ as the average normalized throughput per slot obtained by SU i.

$$\mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}}) \triangleq \frac{T-\tau_i}{T} q(1-P_{f_a}(\tau_i)) \prod_{j\neq i}[1-q(1-P_{f_a}(\tau_j))] \times Pr(\mathcal{H}_0) \quad (5)$$

  From (5) it is straightforward to see that the payoff of SU i is impacted by its own sensing duration choice but also by the decisions of the other SUs on how much they will sense the channel. If $Pr(\mathcal{H}_0) = 0$, i.e. there is always at least one active primary, the SU throughput will always be zero. On the other hand, if $Pr(\mathcal{H}_0) = 1$, there is no need to do sensing and in this case throughput is only limited by contention, i.e. $U_i = q(1-q)^{N-1}$.
- $\mathcal{R} = (r_1, \ldots, r_N)$ represents the throughput requirements for SUs. Consequently, $\forall\, i \in \mathcal{N}$, $\mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}}) = r_i$.

Solutions of the game $\mathcal{G}$ with the property that no individual deviation will happen are referred to as Nash Equilibrium [9]. A formal definition of Nash Equilibrium is given in Definition 1.

Definition 1. *For a game $\mathcal{G} = \{\mathcal{N}, \{\mathcal{A}_i\}_{i\in\mathcal{N}}, \{\mathcal{U}_i\}_{i\in\mathcal{N}}\}$, a strategy profile $\boldsymbol{\tau} = (\tau_i, \boldsymbol{\tau_{-i}})$ is a pure Nash equilibrium if:*

$$\forall\ i \in \mathcal{N}, \forall\ \tau_i^{'} \in \mathcal{A}_i,\ \ \mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}}) \geq \mathcal{U}_i(\tau_i^{'}, \boldsymbol{\tau_{-i}}).$$

Depending on the throughput requirements of SUs, NE might not exist. Even when it does, a SU may end up with the highest achievable throughput at the expense of longer channel sensing duration (i.e. shorter transmission period). One can consider that SUs' objective is to satisfy their throughput requirements instead of considering that their aim is to maximize their own payoff subject to a set of constraints. Thus, every strategy profile of the game where all SUs satisfy their own throughput requirements is considered to be a satisfaction equilibrium [10]. Let $\gamma_i(\boldsymbol{\tau})$ be the set of feasible strategies for a given throughput requirements profile $\mathcal{R}$. Then,

$$\gamma_i(\boldsymbol{\tau}) = \{\tau_i | \mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}}) = r_i,\ \forall i \in \mathcal{N}\}. \tag{6}$$

A formal definition for the concept of SE is provided next.

Definition 2. *For a satisfactory sensing game $\mathcal{G} = \{\mathcal{N}, \{\mathcal{A}_i\}_{i\in\mathcal{N}}, \{\mathcal{U}_i\}_{i\in\mathcal{N}}, \mathcal{R}\}$, a strategy profile $\boldsymbol{\tau} = (\tau_i, \boldsymbol{\tau_{-i}})$ is a SE if:*

$$\forall\ i \in \mathcal{N}, \tau_i \in \gamma_i(\boldsymbol{\tau}).$$

B. Existence and Enumeration of Nash/Satisfaction Equilibria

We will first tackle the existence issue of the NE for the game $\mathcal{G}$. We formulate the following proposition:

Proposition 1. *There exists a unique NE for game $\mathcal{G}$ with no throughput requirements. We denote the corresponding sensing profile by $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_N)$.*

The detailed proof is provided in [8] and is based on the concavity of the payoff function along with the dominance solvability criterion [11]. Both existence and uniqueness of the NE are verified.

Definition 3. *(S-modular game): The strategic form game $\mathcal{G}$ is a supermodular (submodular) game if for every player i:*

i) The strategy space is a compact subset of $\mathbb{R}$;
ii) $\mathcal{U}_i(\tau_i,\boldsymbol{\tau_{-i}})$ is upper semi-continuous in τ_i and continuous in $\boldsymbol{\tau_{-i}}$.
iii) $\mathcal{U}_i$ has increasing (resp. decreasing) differences in $(\tau_i,\boldsymbol{\tau_{-i}})$ i.e., $\frac{\partial^2\mathcal{U}_i(\tau_i,\boldsymbol{\tau_{-i}})}{\partial\tau_i\partial\tau_j} \geq 0$ (resp. $\frac{\partial^2\mathcal{U}_i(\tau_i,\boldsymbol{\tau_{-i}})}{\partial\tau_i\partial\tau_j} \leq 0$).

S-modular games exhibit very nice mathematical properties referred to as strategic complementarities. For super-modular (sub-modular) games, the best response is an increasing (decreasing) function of the other players' strategies. Besides, simple learning algorithms such as iterated best reply dynamics are guaranteed to converge to one of the extreme equilibria. The satisfactory sensing game $\mathcal{G}$ is S-modular and switches modularity over the PU frame duration T.

Proposition 2. *There exists a unique $\mu_i \in (0, T]$ such as $\mathcal{G}$ is submodular over $(0, \mu_i]$ and supermoduler over $[\mu_i, T]$ where $\boldsymbol{\mu}$ is the NE profile of the game $\mathcal{G}$.*

Proof: Let us compute the mixed second-order derivative of the payoff $\mathcal{U}_i(\tau_i,\boldsymbol{\tau_{-i}})$ for SU i with respect to its own strategy τ_i and the strategy of another SU τ_j.

$$\frac{\partial^2\mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}})}{\partial\tau_i\partial\tau_j} = -\frac{B\mathrm{e}^{-\frac{A^2\tau_j}{2}}Aq^2}{8T\pi^{3/2}\sqrt{\tau_j\tau_i}} \times \frac{C}{\sqrt{2}} \tag{7}$$

where:

$$C \triangleq \left(A\sqrt{2}\,(T-\tau_i)\,\mathrm{e}^{-\frac{A^2\tau_i}{2}} + 2\sqrt{\pi}\left(\mathrm{erfc}\left(\frac{A\sqrt{\tau_i}}{\sqrt{2}}\right) - 2\right)\sqrt{\tau_i}\right)$$

and

$$B \triangleq \prod_{k\neq i,j}\left[1 - q(1 - P_{f_a}(\tau_k))\right] \tag{8}$$

When the sensing duration equals the SU's frame duration T, $\frac{\partial^2\mathcal{U}_i(\tau_i,\boldsymbol{\tau_{-i}})}{\partial\tau_i\partial\tau_j}$ has the following value:

$$-\frac{B\mathrm{e}^{-\frac{A^2\tau_j}{2}}Aq^2\sqrt{2}\left(\mathrm{erfc}\left(\frac{A\sqrt{T}}{\sqrt{2}}\right) - 2\right)}{8T\sqrt{\tau_j\pi}} \tag{9}$$

We notice that $\frac{\partial^2\mathcal{U}_i(\tau_i,\boldsymbol{\tau_{-i}})}{\partial\tau_i\partial\tau_j}|_{\tau_i=T} \geq 0$.

For very small values of τ_i we have:

$$\lim_{\tau_i\to 0}\frac{\partial^2\mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}})}{\partial\tau_i\partial\tau_j} = \lim_{\tau_i\to 0} -\frac{B\mathrm{e}^{-\frac{A^2\tau_j}{2}}A^2q^2}{8\pi\sqrt{\tau_i\tau_j}} \leq 0 \tag{10}$$

We inspect the monotonicity of the mixed second-order partial derivative of $\mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}})$:

$$\frac{\partial}{\partial\tau_i}\left[\frac{\partial^2\mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}})}{\partial\tau_i\partial\tau_j}\right] = \frac{B(Aq)^2\mathrm{e}^{-\frac{A^2}{2}(\tau_i+\tau_j)}\left(A^2\tau_i(T-\tau_i) + T + 3\,\tau_i\right)}{16T\pi\,{\tau_i}^{3/2}\sqrt{\tau_j}} \tag{11}$$

The first-order partial derivative of $\frac{\partial^2\mathcal{U}_i(\tau_i,\boldsymbol{\tau_{-i}})}{\partial\tau_i\partial\tau_j}$ is positive. Consequently, the mixed second-order derivative of $\mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}})$ is non-decreasing in τ_i.

By continuity and monotonicity of the function $\frac{\partial^2\mathcal{U}_i(\tau_i,\boldsymbol{\tau_{-i}})}{\partial\tau_i\partial\tau_j}$ and from equations (9) and (10), we conclude that there exists a unique $\mu_i \in (0, T]$ such that:

- $\mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}})$ is sub-modular on $(0, \mu_i]$
- $\mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}})$ is super-modular on $[\mu_i, T]$

To prove that $\boldsymbol{\mu}$ coincides with the NE of the game $\mathcal{G}$, we state the first-order optimality condition:

$$\frac{\partial\,\mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}})}{\partial\,\tau_i} = 0$$

$$\iff \frac{\left(q(\mathrm{erfc}\left(A\sqrt{\frac{\tau_j}{2}}\right) - 2) + 2\right)qB}{8T\sqrt{\pi\tau_i}}C = 0$$

By noting that $\left(q(\text{erfc}\left(A\sqrt{\frac{\tau_j}{2}}\right) - 2) + 2\right) > 0$, we have:

$$\frac{\partial\, \mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}})}{\partial\, \tau_i}|_{\tau_i=\mu_i} = 0$$

$$\iff \frac{\partial^2 \mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}})}{\partial \tau_i \partial \tau_j}|_{\tau_i=\mu_i} = 0 \iff C = 0,$$

where:

$$\mu_i = \arg\max_{\tau_i} \mathcal{U}_i(\tau_i, \boldsymbol{\tau_{-i}}) \quad (12)$$

■

We proceed now to the enumeration of existing satisfactory equilibria by taking into account the mathematical properties of the payoff functions. We will use the fact that the game is S-modular and quasi-concave to enumerate its satisfaction equilibria, as stated in Theorem 1.

Theorem 1. *The satisfactory sensing game $\mathcal{G}$ has either zero, one, or two satisfaction equilibria.*

Proof: Without loss of generality, we restrict ourselves to the satisfaction game played by two SUs i and j. The same reasoning holds for the general case when $N > 2$, since no specific assumption is made about i and j. Let $\mathcal{U}_i^{max}$ denote the maximum residual capacity of the channel for a given SU. In order for the game $\mathcal{G}$ to possess at least one equilibrium, the following condition on the sum of required throughputs must hold:

$$\sum_{i\in\mathcal{N}} r_i \leq \sum_{i\in\mathcal{N}} U_i^{max}. \quad (13)$$

If condition (13) holds, by the concavity of the payoff function for each player $i \in \mathcal{N}$ w.r.t its own strategy (sensing duration τ_i), $\mathcal{U}_i(\tau_i, \tau_j) = r_i$ admits:

- One solution if $r_i = \mathcal{U}_i^{max}$ or $r_i < \mathcal{U}_i(0, \tau_j), \forall\, \tau_j \in [0, T]$.
- Two solutions if $\mathcal{U}_i(0, \tau_j) \leq r_i < \mathcal{U}_i^{max}, \forall\, \tau_j \in [0, T]$.

The two possible cases are illustrated in Fig. 1 along with the extreme payoffs $\mathcal{U}_i(\tau_i, 0)$ and $\mathcal{U}_i(\tau_i, T)$. Since $\mathcal{U}_i(\tau_i, \tau_j)$ is strictly increasing in τ_j, the payoff $\mathcal{U}_i(\tau_i, \tau_j)$ will be bounded by the extreme payoffs.

The first case is straightforward to prove. We investigate the second case where every user i has two possible choices τ_i^1 and τ_i^2 with $\tau_i^1 < \tau_i^2$. All the possible combinations of size M are potential strategies. We argue that among all the possible combinations, only two solutions, namely: $\underline{\boldsymbol{\tau}} = (\tau_1^1, \ldots, \tau_i^1, \tau_M^1)$ and $\overline{\boldsymbol{\tau}} = (\tau_1^2, \ldots, \tau_i^2, \tau_M^2)$ are satisfaction equilibria, satisfying the following inequality:

$$\underline{\boldsymbol{\tau}} \prec \boldsymbol{\mu} \prec \overline{\boldsymbol{\tau}} \quad (14)$$

with $\prec$ being the component-wise order operator.

By Proposition 2, the proposed game has strategic complementarities. Consequently, if player i decides to play τ_i^2 thus increasing its sensing duration, then the interest of the others is to increase their strategies, as the game is supermodular on $[\mu_i, T]$. Conversely, as the game is submodular on $(0, \mu_i]$, if one player i decides to play τ_i^1 thus decreasing its sensing duration, then the interest of the others is to decrease their strategies as the game has decreasing differences. We conclude that the satisfactory sensing game $\mathcal{G}$ admits two equilibria $\{\underline{\tau}, \overline{\tau}\}$ when every SU has two sensing duration choices. ■

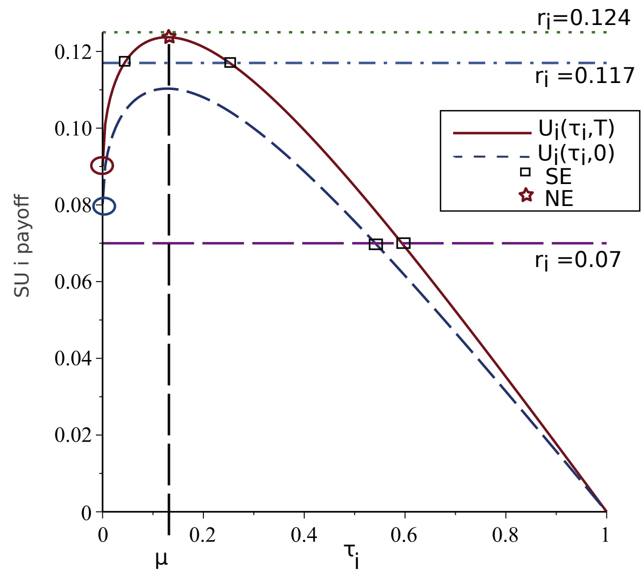

Fig. 1. Number of SEs as a function of the required throughput (T=1).

C. Equilibrium selection

As mentioned previously, the satisfactory game $\mathcal{G}$ has either zero, one or two satisfaction equilibria. This gives rise to the problem of selecting the optimal SE when there are two of them. Indeed, it may be tempting to always choose the smallest SE to maximize the transmission duration. Nevertheless, in the imperfect sensing scenario, this could be very degrading to the primary users if their detection fail.

The IEEE 802.22 standard defines thresholds for the false-alarm probability ($P_{fa} = Pr(\mathcal{H}_1|\mathcal{H}_0)$) and miss-detection probability ($P_{md} = Pr(\mathcal{H}_0|\mathcal{H}_1)$) [12]. Typical values range from 0.01 to 0.1 for P_{fa} and from 0.05 to 0.1 for P_{md}. To simplify the selection of the SE we propose to take into consideration the value of P_{md}, estimated over a window of previous slots. The intuition behind this approach is that if P_{md} is very small, then it is safe to sense the channel for a small duration (the smallest SE) and consequently achieve a larger transmission time without degrading the primary user' performance. In contrast, if P_{md} is high, then the SUs will sense the channel for a longer duration (the largest SE) and will transmit for a shorter period of time in order to not perturb primary users.

D. Symmetric game analysis

We shift our attention to the symmetric game where all SUs sense the channel for the same duration τ (scalar value). The payoff for each SU simplifies to the following:

$$\mathcal{U}_i(\tau,\ldots,\tau)=\frac{T-\tau}{T}q(1-P_{f_a}(\tau))[1-q(1-P_{f_a}(\tau))]^{(N-1)} \times Pr(\mathcal{H}_0) \quad (15)$$

To find the optimal access probability q, we formulate the first-order optimality condition:

$$\frac{d\,\mathcal{U}_i(\tau,\ldots,\tau)}{d\,q}=-\frac{(T-\tau)\,(1+n\,(-1+f_a)\,q)}{T\,(1+(-1+f_a)\,q)} \times (1+(-1+f_a)\,q)^{n-1}\,(-1+f_a)\,Pr(\mathcal{H}_0)=0 \quad (16)$$

This condition simplifies to:

$$q^*=\frac{1}{n\,(1-f_a)}. \quad (17)$$

Note that when the false-alarm probability $P_{f_a}=0$, we arrive at the maximal achievable throughput per user for slotted aloha: $\frac{1}{N}$. Also, the optimal access probability q^* is independent of the channel idleness probability $Pr(\mathcal{H}_0)$. We illustrate in Fig. 2 the evolution of the optimal access probability q^* for different values of τ in the case of seven secondary users.

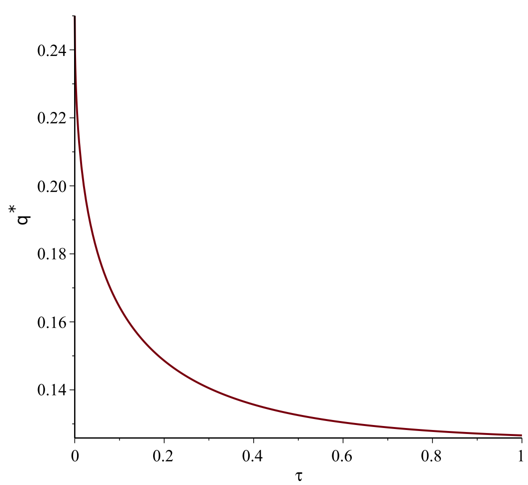

Fig. 2. Optimal access probability vs normalized sensing duration ($T=1$).

From Fig. 2 we conclude that as the sensing duration increases, SUs will be less tempted to access the channel since the transmission phase duration $T-\tau$ will decrease.

IV. Distributed Algorithm For Learning Satisfaction Equilibria

In centralized learning schemes, each player selects his strategies sequentially. In each time slot, a player selects a strategy that represents the best response (BR) to the strategies chosen by the other players in the previous time slot. BR-based centralized algorithms such as best reply dynamics give nice convergence results for a particular class of games, including submodular/supermodular games. However, they require that each player knows what strategies the other players are taking. In our case, the player (SU) may not be aware of the sensing duration chosen by other SUs. Thus, a learning mechanism that relaxes this assumption is needed. Players observe only their own payoff, and own strategies; the strategies of the others are not needed. We propose a distributed learning algorithm based on FDTPA [13] to learn the satisfactory equilibria.

The main ideas of SELA are the following :

- Each SU can observe the sucess/failure of its own transmission attempt.
- SU can learn its effective throughput based on the observation result.
- The SU computes the deviation of its current throughput from its own demand (r_i). Then, adjusts the sensing durations to decrease this error to zero.
- On convergence, each SU's effective throughput satisfies its own requirement.

Let $\mathbb{1}_{success}^{\{q,P_{f_a},\mathcal{H}_0,i\}}$ be an indicator function that takes the value of one when the transmission for SU i is successful, given access probability q, false-alarm probability P_{f_a} and channel idleness probability $Pr(\mathcal{H}_0)$. The Satisfactory Equilibrium Learning Algorithm (SELA) is illustrated in Algorithm 1.

Algorithm 1: Satisfactory Equilibrium Learning Algorithm

Data:
H: Learning horizon;
$\mathcal{N}$: Set of SUs;
q: Channel access probability;
δ_i: Learning step size for SU i;
Result: Satisfaction Equilibrium profile (τ_i,τ_{-i}), $i\in\mathcal{N}$

Initialization: Each SU i, initializes $\tau_{i,0}$;
while $k\leq H$ **do**
  foreach *player i = 1,2,…,N* **do**
    $\mathcal{U}_i^{k+1}:=\mathcal{U}_i^k+\delta_i^{k+1}\times\mathbb{1}_{success}^{\{q,P_{f_a},\mathcal{H}_0,i\}}\times\frac{T-\tau_i^k}{T}$
    $\tau_i^{k+1}:=\tau_i^k+\delta_i^{k+1}(r_i-\mathcal{U}_i^{k+1})$
  end
  $k:=k+1$;
end

V. Numerical Results

For our simulations, we set the energy detection parameters such that: $(\frac{\epsilon}{\sigma^2}-1)\sqrt{f_s}=7$. To cope with the transmission randomness induced by the slotted aloha channel access and false-alarm probabilities, the simulations were repeated 100 times and the results averaged. We consider the following scenarios:

- Two SUs (SU_1,SU_2) with symmetric throughput requirements $(0.1,0.1)$. The channel access probability q takes the values $\{0.2,0.21,0.22\}$ and the probability of the channel being idle is $Pr(\mathcal{H}_0)=0.95$.

- Three SUs (SU_1, SU_2, SU_3) with asymmetric throughput requirements $(0.076, 0.08, 0.09)$ and the SUs payoffs are the general payoffs conditioned on the channel being idle (average throughput normalized by $Pr(\mathcal{H}_0)$).

First, we numerically compute the normalized SE and NE for the first scenario when $q = 0.2$. The results are depicted in Fig. 3.

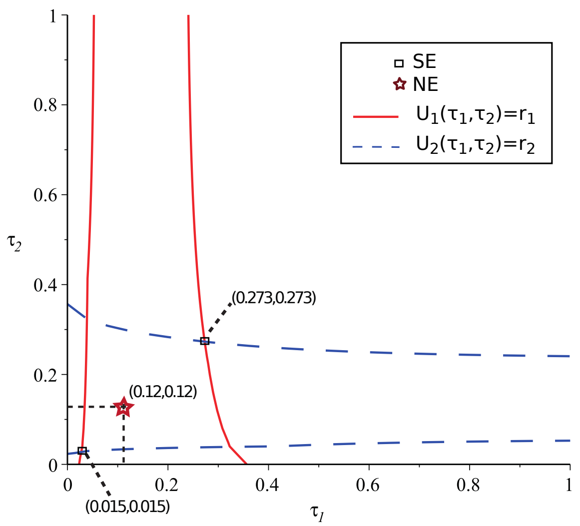

Fig. 3. Sensing durations τ_1 and τ_2 when $r_1 = r_2 = 0.1$, $T = 1$ and $q = 0.2$.

SELA was run with the throughput requirements of the first simulation scenario. The SE when $q = 0.2, 0.21$ and 0.22 (to which SELA converges) are illustrated in Fig.4 and Fig.5. We first notice that for $q = 0.2$, SE discovered by SELA are very close to those computed numerically. For the largest SE, as q increases the required sensing duration grows proportionally. Whereas for the smallest SE, the opposite behavior is obtained and the sensing duration decreases when q increases.

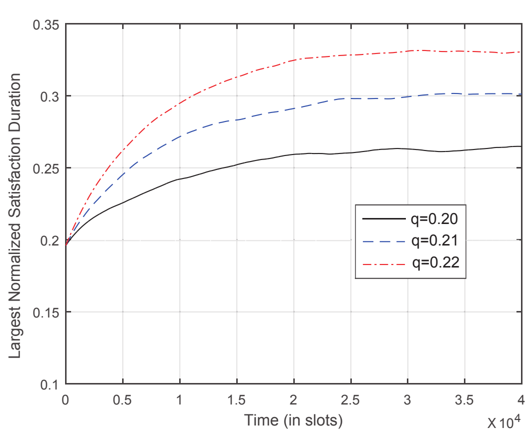

Fig. 4. Normalized satisfaction duration for different q values ($r_1 = 0.1$, $r_2 = 0.1$).

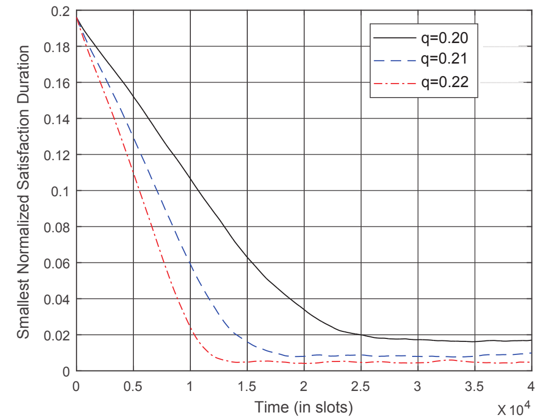

Fig. 5. Normalized satisfaction duration for different q values ($r_1 = r_2 = 0.1$).

The largest SE provides better protection to the PU. Indeed, since the SUs will sense the channel for a longer duration, their probability of miss-detection P_{md} will decrease. Sensing for a longer time will also result in reducing the false-alarm probability P_{f_a}. When the sensing capabilities of the SUs are very good and the standard defined threshold on P_{md} very small, the smallest equilibrium will be interesting to the SUs as it maximizes the transmission period duration.

In the second scenario, we consider the payoff conditioned on the channel being idle. We run SELA with the corresponding throughput requirements and obtained convergence for the two SE. The largest and smallest SE are depicted in Fig. 6 and Fig. 7, respectively.

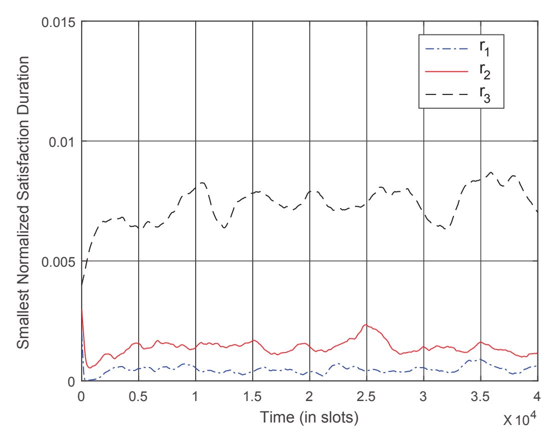

Fig. 6. Normalized Satisfaction Duration $r_1 = 0.076$, $r_2 = 0.08$, $r_3 = 0.09$

In Fig. 6 we notice that at the smallest SE, the sensing duration is an increasing function of the required throughput. As the SUs throughput requirements grow, SUs should listen more to the channel. In contrast, for the largest satisfaction equilibrium illustrated in Fig. 7, the sensing duration is a decreasing function of the required throughput. SUs with higher throughput requirements will sense the channel less often than SUs with smaller requirements.

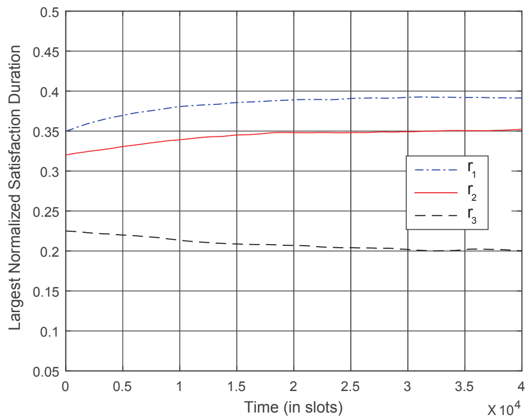

Fig. 7. Normalized SE Duration ($r_1 = 0.076$, $r_2 = 0.08$, $r_3 = 0.09$)

Only 2×10^4 iterations are required for SELA to converge to the satisfaction equilibria. Fig. 8 illustrates the evolution of the SUs' normalized throughput as they discover their satisfaction equilibria. As SELA converges to SEs, the obtained throughput becomes identical to the SUs requirements.

One interesting feature of having two SEs is that we could use the smallest one to enhance the underlay access in cognitive networks by sensing for a short duration before attempting to communicate. The largest SE will be used in traditional overlay CRNs to define the sensing period.

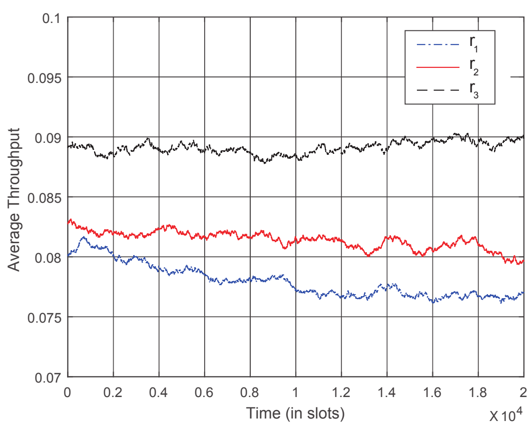

Fig. 8. Normalized throughput at SE ($r_1 = 0.076$, $r_2 = 0.08$, $r_3 = 0.09$).

VI. Concluding remarks

This paper introduced a distributed mechanism for determining the duration of the sensing period in CRNs. We modeled the problem as a satisfactory game with a set of throughput requirements for SUs. First, we provided existence and uniqueness proofs for the Nash Equilibrium and then demonstrated that the satisfactory game has either zero, one or two Satisfaction Equilibria. Then, we proposed a distributed Satisfaction Equilibrium Learning Algorithm (SELA) and conducted simulations and numerical investigations to corroborate our theoretical results. We also presented a satisfaction equilibrium selection approach based on the miss-detection probability.

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