

Stochastic Guard-Band-Aware Channel Assignment With Bonding and Aggregation for DSA Networks

Mohammad J. Abdel-Rahman, *Member, IEEE*, and Marwan Krunz, *Fellow, IEEE*

Abstract—Fading and shadowing along with the primary user dynamics make channel quality in dynamic spectrum access networks uncertain. Furthermore, the imperfect design of filters and amplifiers in wireless devices motivates the need for guard-bands (GBs) to prevent adjacent-channel interference. In this paper, we develop novel stochastic GB-aware sequential and batch channel assignment schemes that aim at maximizing the spectrum efficiency. In line with recent IEEE 802.11 and LTE standards, our schemes support bonding and aggregation. We propose two assignment models for each of the sequential and batch schemes: a static single-stage and an adaptive two-stage. In the static model, channel assignment is performed once such that the rate demands are probabilistically met. The adaptive model is a two-stage model, where the initial assignment may be corrected once uncertainties are partially revealed. We refer to our formulations of the sequential and batch static assignments as *chance-constrained stochastic subset-sum problem (CSSP)* and *chance-constrained stochastic multiple subset-sum problem (CMSSP)*, respectively. Moreover, we develop stochastic formulations for the sequential and batch adaptive assignments, which we refer to as *two-stage CSSP with recourse (CSSPR)* and *two-stage CMSSP with recourse (CMSSPR)*, respectively. Finally, we present computationally efficient simplified versions of CSSP and CSSPR with near-optimal performance.

Index Terms—Channel assignment, dynamic spectrum access, guard bands, multiple subset-sum problem, spectrum efficiency, stochastic optimization.

I. INTRODUCTION

MOTIVATED by the need for more efficient utilization of the licensed spectrum, and supported by recent regulatory polices (e.g., [2]), significant research has been conducted on dynamic spectrum access (DSA) networks. In a DSA system, secondary users (SUs) access the available spectrum in a dynamic and opportunistic fashion, without interfering with co-located incumbent users, i.e., primary users (PUs).

Manuscript received March 26, 2014; revised September 24, 2014 and January 15, 2015; accepted February 7, 2015. This work was supported by NPRP grant # NPRP 4-1034-2-385 from the Qatar National Research Fund (a member of Qatar Foundation). The statements made herein are solely the responsibility of the authors. A preliminary version of this paper was presented at IEEE GLOBECOM, December 2013. The associate editor coordinating the review of this paper and approving it for publication was F. Gao.

M. J. Abdel-Rahman was with the University of Arizona, Tucson, AZ 85721 USA. He is now with the Department of Electrical and Computer Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061 USA (e-mail: mjabdelrahman@email.arizona.edu).

M. Krunz is with the Department of Electrical and Computer Engineering, University of Arizona, Tucson, AZ 85721 USA (e-mail: krunz@email.arizona.edu).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TWC.2015.2413964

Channel quality in DSA networks is uncertain. This is partially due to inherent multi-path fading and shadowing, and partially to the unpredictability of PU activity. Recently, the FCC advocated using a database (DB) to facilitate DSA. According to this approach, an SU acquires the set of available channels in its geographical area through a centralized DB. The DB is mostly concerned with SU-to-PU interference, but not PU-to-SU interference [3]. It declares a channel to be available at a given location if the PU signal transmitted over this channel cannot be successfully decoded by a PU receiver at that location. As indicated in [4], the decodability threshold for a digital TV signal is ~ 15 dB, which is ~ 32 times stronger than the noise level. Therefore, even if a channel is declared available by the DB, this does not mean that it is completely clean, and it can still have a substantial “pollution” due to PU transmissions. In other words, PU dynamics causes the channel quality of a DSA network to be uncertain.

In addition to channel uncertainty, we consider two aspects: (i) accounting for adjacent-channel interference (ACI), and (ii) supporting channel bonding and aggregation. ACI is a form of power leakage that is attributed to imperfect filters and amplifiers in the radio device. The harmful impact of ACI on the throughput of IEEE 802.11a and IEEE 802.11n networks was demonstrated in [5] and [6], respectively. Most channel assignment algorithms in the literature do not account for ACI. To mitigate ACI, *guard-bands* (GBs) are needed between adjacent channels that belong to different links. Introducing GBs constrains spectrum efficiency. We aim at maximizing spectrum efficiency while mitigating ACI.

Channel bonding and aggregation enables the support of applications with high rate demands. Channel bonding refers to the bundling of adjacent channels, which can then be treated as a single frequency “block” whose data rate is approximately the sum of the data rates of the individual channels. Channel bonding has been adopted in recent IEEE 802.11n and IEEE 802.11ac standards [7]–[11]. It can be extended to non-adjacent channels, a concept referred to as *channel aggregation*. For example, LTE-Advanced employs channel aggregation techniques, allowing 4G mobile operators to aggregate spectrum from non-adjacent bands to support links with high demands [12]. Several resource allocation schemes with channel bonding (e.g., [13]–[15]) and aggregation (e.g., [16]–[18]) were proposed in the literature. However, none of these schemes account for ACI through GBs.

Recently, Uyanik *et al.* [19] proposed a GB-aware (GBA) channel assignment scheme that achieves optimal spectrum efficiency and supports channel bonding and aggregation.

However, the transmission rates of various channels were assumed to be deterministically known. Our objective in this paper is to design GBA channel assignment schemes for DSA networks, where channel quality is uncertain. It has been shown in [19], [20] that to attain optimal spectrum efficiency, channels need to be assigned on a per-block basis (as will be explained in Section II). Given this, the channel assignment problem in [19] can be restated as follows: Given the set of idle frequency blocks, obtain a combination of idle frequency blocks that either satisfies the link demand or achieves the nearest rate to this demand. This is exactly the subset-sum problem (SSP) [21].¹

A few stochastic SSP formulations were proposed in the literature to tackle the SSP under the uncertainty of the item weights and/or the knapsack capacity. Existing stochastic SSP formulations are not suitable for our stochastic GBA channel assignment problems. Therefore, using stochastic optimization theory, in this paper we develop new stochastic SSP formulations. To the best of our knowledge, none of the existing channel assignment schemes in the literature supports channel bonding, aggregation, and GBs, while simultaneously accounting for the channel quality uncertainty.

Our Contributions: To account for the uncertainty in channel quality, we develop novel stochastic GBA channel assignment schemes that maximize the spectrum efficiency and support channel bonding and aggregation. We propose both sequential (i.e., per-link) as well as batch (multi-link) channel assignment schemes. For each, we formulate two stochastic channel assignment models: *a static single-stage* and *an adaptive two-stage*. In the static model, channel assignment is performed once such that the given rate demands are met with a probability greater than a certain threshold. This model is appropriate for distributed systems that do not have a centralized spectrum manager. The adaptive model is a two-stage assignment model, where the initial assignment may be corrected once the uncertainties are *partially* revealed. This model is suitable for systems with a centralized spectrum manager.

We develop two stochastic SSP formulations for the sequential and batch static GBA channel assignment problems. We refer to these formulations as *chance-constrained stochastic SSP (CSSP)* and *chance-constrained stochastic multiple SSP (CMSSP)*, respectively. The “chance constraint” is introduced to restrict the probability of under-satisfying the link demand. We solve CSSP and CMSSP by deriving their deterministic equivalent formulations, which turn out to be binary integer linear programs (BILPs). Furthermore, we propose two formulations for the sequential and batch adaptive GBA channel assignment problems. We refer to these formulations as *two-stage CSSP with recourse (CSSPR)* and *two-stage CMSSP with recourse (CMSSPR)*, respectively. The second stage in CSSPR and CMSSPR is introduced to prevent link over-satisfaction, and also to improve the rate of the under-satisfied links in the first stage. Similar to CSSP and CMSSP, we solve CSSPR and CMSSPR by deriving their deterministic equivalent formulations, which also turn out to be BILPs. Finally, we develop computationally efficient simplified versions of CSSP and CSSPR,

¹Note that if aggregation is not allowed, the assignment problem becomes much easier, and it can be efficiently solved using the Hungarian method.

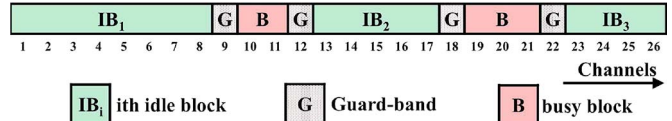


Fig. 1. Example of a spectrum status ($M = 26$ and $N = 3$). Channels 10–11 and 19–21 are busy. Two GBs are allocated around each block of busy channels, one on each side. After allocating the GBs, three blocks of idle channels are left: channels 1–8, 13–17, and 23–26.

and compare their performance with the exact deterministic equivalent formulations.

Paper Organization: The rest of the paper is organized as follows. In Section II, we provide the system model followed by the problem statement. The sequential static and adaptive channel assignment problems are formulated in Section III-A, and reformulated as BILPs in Section III-B. In Section IV-A, we formulate the batch static and adaptive channel assignment problems. The batch assignment problems are reformulated as BILPs in Section IV-B. In Section V, we present our numerical results. We conclude the paper in Section VI and provide directions for future research.

II. PROBLEM STATEMENT

Consider a DSA network² that consists of L links and operates over M licensed channels. An idle (i.e., PU free) channel can be used as a GB if it is adjacent to a busy channel, or as a data channel otherwise. A channel is considered busy if it is occupied by a PU or previously admitted SU. Assume that the current set of idle channels are grouped into N frequency blocks (after reserving the needed GBs). Each block consists of contiguous idle channels. Let $\mathcal{N} \stackrel{\text{def}}{=} \{1, 2, \dots, N\}$ and let $\tilde{R}_i, i \in \mathcal{N}$, be a random variable representing the data rate supported by the i th idle frequency block,³ denoted by IB_i . Let $\mathcal{L} \stackrel{\text{def}}{=} \{1, 2, \dots, L\}$ and let $d_j, j \in \mathcal{L}$, be the rate demand of link j in Mbps. Given the current spectrum status, i.e., the state of each of the M licensed channels, our objective is to satisfy the rate demands of the L links while maximizing the spectrum efficiency, defined as the fraction of the available spectrum that can be used for data communications. Fig. 1 shows an example of a spectrum status. The assignment that maximizes spectrum efficiency is the one that minimizes the number of introduced GBs.

Theorem 1: To maximize spectrum efficiency, channels need to be assigned on a per-block basis instead of per-channel basis [19].

Proof: See [19]. ■

In this paper, we study the following two problems:

Problem 1 (Sequential Assignment): Given an arbitrary link with a rate demand d Mbps, the current status of the M channels, and the rate distributions of the idle channels, find the

²The treatment in this paper applies to any wireless network. Considering a DSA network only affects the calculation of the rate distribution (which depends on the PU activity pattern). Because PU-to-SU interference is usually overlooked, channel-quality uncertainty is particularly important in DSA networks.

³The rate distribution of an idle frequency block is obtained from the rate distribution of the idle channels that constitute this block.

optimal GBA channel assignment for this link that maximizes the spectrum efficiency while satisfying the demand d .

Problem 2 (Batch Assignment): Given the set of links \mathcal{L} and their associated rate demands, the current status of the M channels, and the rate distributions of the idle channels, find the optimal GBA channel assignment that maximizes the spectrum efficiency while satisfying the link demands.

As mentioned earlier, our proposed assignment schemes support channel bonding and aggregation. In our formulations, we do not impose a limit on the total bonded/aggregated bandwidth. Although current wireless systems impose such a limit, there are several ongoing efforts to develop systems that support wide-band aggregation. The trend in the current IEEE 802.11/LTE standards is to increase the supported data rate by enabling bonding/aggregation of more channels.

We study Problems 1 and 2 considering two different system models. In the first model, channel block-assignment is performed once and cannot be corrected after the actual transmission rates become known to the assignment algorithm. This corresponds to a network setup where channel assignment is performed for each link independent of other links. The second model allows for adjusting the block assignment after the randomness is partially revealed, when nodes start communicating over the assigned blocks. According to the second model, the additional assigned blocks (if any) can be taken from over-satisfied links and reassigned to under-satisfied links. This dynamic assignment approach increases the number of admitted links in a resource-constrained network, operating under channel uncertainty.

III. SEQUENTIAL ASSIGNMENT

A. Problem Formulation

In this section, we formulate the sequential (per-link) channel block-assignment problem. Because the rates of idle blocks are random, assigning blocks based on their expected rates only may result in link under-satisfaction (when the actual rates are lower than their expected values) or link over-satisfaction (when the actual rates are higher than their expected values). To handle the uncertainty associated with these rates, we propose two different models based on stochastic programming techniques. The first model introduces a ‘‘chance constraint’’ to limit the likelihood of under-satisfaction, whereas the second model goes a step further, allowing for some corrective action in case of over-satisfaction.

1) *Static Single-Stage Assignment:* Considering a single link, in this section we formulate the channel block-assignment problem assuming the static (non-corrective) model. In this model, the assignment is performed once, and correcting the assignment after observing the actual block rates is not allowed. As mentioned earlier, the optimal channel block-assignment problem that maximizes spectrum efficiency was formulated in [19] as an SSP, assuming deterministic channel rates. When the channel rates are random, the feasible region of the assignment problem becomes uncertain. Different stochastic optimization approaches have been proposed in the literature to deal with the uncertainty of the feasibility region of an opti-

mization problem [22]. In here, we adopt a ‘‘chance constraint approach.’’

We formulate the static stochastic assignment problem under rates uncertainty as a chance-constrained stochastic SSP (CSSP) with a discrete distribution. Let $x_i, i \in \mathcal{N}$, be a binary variable; $x_i = 1$ if the i th frequency block is to be assigned, and zero otherwise. Then, the CSSP formulation is given by:

Problem CSSP:

$$\text{minimize}_{\{x_i, i \in \mathcal{N}\}} \sum_{i=1}^N \mu_i x_i \quad (1)$$

$$\text{subject to} \quad \Pr \left\{ \sum_{i=1}^N \tilde{R}_i x_i \geq d \right\} \geq \beta \quad (2)$$

$$x_i \in \{0, 1\}, \forall i \in \mathcal{N} \quad (3)$$

where $\mu_i \stackrel{\text{def}}{=} \mathbb{E}[\tilde{R}_i]$ and $\beta \in (0, 1]$ is a given probability. The objective (1) is to minimize the total expected rate of the assigned blocks, and the chance constraint (2) enforces satisfying the link demand with probability $\geq \beta$. While the chance constraint probabilistically accounts for link under-satisfaction, it does not hedge against the problem of link over-satisfaction. In a resource-constrained network with multiple links operating in parallel, over-satisfying one link may result in under-satisfying other links in the network.

2) *Adaptive Two-Stage Assignment:* The adaptive (corrective) channel block-assignment problem is formulated in this section. The frequency blocks are initially assigned based on their expected rates while satisfying the chance constraint. After observing the actual rates supported by the assigned blocks, additional blocks (if any) are released and returned to the spectrum manager. The returned blocks can then be used by other links.

Let $\mathbf{x} = (x_1, x_2, \dots, x_N)$ and $\tilde{\mathbf{R}} = (\tilde{R}_1, \tilde{R}_2, \dots, \tilde{R}_N)$. Then, the adaptive stochastic assignment problem is formulated as a two-stage CSSP with recourse (CSSPR), as follows:

Problem CSSPR:

$$\text{minimize}_{\{x_i, i \in \mathcal{N}\}} \left\{ \sum_{i=1}^N \mu_i x_i + \mathbb{E} \left[h(\mathbf{x}, \tilde{\mathbf{R}}) \right] \right\} \quad (4)$$

$$\text{subject to} \quad \Pr \left\{ \sum_{i=1}^N \tilde{R}_i x_i \geq d \right\} \geq \beta \quad (5)$$

$$x_i \in \{0, 1\}, \forall i \in \mathcal{N} \quad (6)$$

where $h(\mathbf{x}, \tilde{\mathbf{R}})$ is the optimal value of the second-stage problem, which is given by:

$$\text{minimize}_{\{y_i, i \in \mathcal{N}\}} \left\{ - \sum_{i=1}^N \alpha_i \tilde{R}_i y_i \right\} \quad (7)$$

$$\text{subject to} \quad y_i \leq x_i, \forall i \in \mathcal{N} \quad (8)$$

$$\sum_{k=1}^N (x_k - y_k) \tilde{R}_k \geq d y_i, \forall i \in \mathcal{N} \quad (9)$$

$$y_i \in \{0, 1\}, \forall i \in \mathcal{N}. \quad (10)$$

$\alpha_i \in [0, 1), i \in \mathcal{N}$, is a discount factor. The second-stage decision variable $y_i, i \in \mathcal{N}$, equals one if block i is removed (i.e., unassigned after it was previously assigned) and zero otherwise. The objective of the second-stage problem in (7) is to maximize the total rate of the released extra blocks. We assume that the rate of any block $IB_i, i \in \mathcal{N}$, at the second-stage (i.e., when the block is released after it was previously assigned) is strictly smaller than its first-stage rate, i.e., $\alpha_i < 1$. This way, the later the block is used by a link the smaller the rate this block can support. Setting α_i to be strictly less than one for all $i \in \mathcal{N}$ avoids having an undesirable aggressive assignment (when $\alpha_i = 1, \forall i \in \mathcal{N}$), in which a single link reserves all available blocks and then releases the additional ones. This approach is undesirable because all resources will be allocated to one link first, then all the surplus blocks will be allocated to another link, and so on. Constraint (8) enforces that only blocks that have been assigned in the first stage may be removed, and constraint (9) ensures that a block can be removed only when the first-stage assignment has led to an over-satisfaction, and that the link demand remains satisfied after the removal. As in CSSP, a chance constraint is introduced in the first stage to restrict the probability of under-satisfaction.

We note that CSSPR has a relatively complete recourse, i.e., for every feasible first-stage decision x_i that satisfies (5) and (6), there exists a feasible solution to the second-stage problem under each scenario $\omega \in \Omega$, where Ω is the set of ‘‘scenarios,’’ various realizations of the rates of various blocks. For example, $y_i = 0, \forall i \in \mathcal{N}$, is always a feasible solution to the second-stage problem.

B. Problem Reformulation and Solution Approach

Our approach for solving the CSSP and CSSPR formulations is to derive their deterministic equivalent programs (DEPs). The DEP is an equivalent reformulation of the original stochastic program, but contains only deterministic variables [22]. In this section, we first present the DEPs of CSSP and CSSPR. Because of their high complexity, in this section we also propose simplified versions of the CSSP and CSSPR formulations, which we refer to as CSSP-modified and CSSPR-modified, respectively.

1) Static Single-Stage Assignment:

a) *DEP*: To obtain the DEP of CSSP, we need to reformulate the chance constraint (2), so that it does not include the probability term or the random variables $\tilde{R}_i, i \in \mathcal{N}$. Let $p^{(\omega)}$ be the probability of scenario $\omega \in \Omega$. $p^{(\omega)}$ is considered as an input to our assignment problems. To reformulate the chance constraint, we will introduce a binary variable $u^{(\omega)}$ for each scenario $\omega \in \Omega$; $u^{(\omega)} = 0$ if the block assignment needs to satisfy the demand d under scenario ω , and one otherwise. Then, the chance constraint (2) is equivalent to the following two constraints:

$$\sum_{i=1}^N R_i^{(\omega)} x_i \geq d (1 - u^{(\omega)}), \quad \forall \omega \in \Omega \quad (11)$$

$$\sum_{\omega \in \Omega} p^{(\omega)} u^{(\omega)} \leq 1 - \beta. \quad (12)$$

TABLE I
COMPLEXITY OF VARIOUS SSP SOLUTIONS

Algorithm	Complexity
Dynamic programming (DP)-based exact	$\mathcal{O}(Nd)$
Pseudo-polynomial ϵ -approximate	$\mathcal{O}(\frac{1}{\epsilon} N^2 \ln d)$
Polynomial-time greedy	$\mathcal{O}(N \log N)$

For each scenario $\omega \in \Omega$, if $u^{(\omega)} = 0$ then (11) reduces to $\sum_{i=1}^N R_i^{(\omega)} x_i \geq d$ (i.e., the link demand d is satisfied under scenario ω). On the other hand, if $u^{(\omega)} = 1$, then (11) reduces to $\sum_{i=1}^N R_i^{(\omega)} x_i \geq 0$ which is a redundant constraint since the data rates of all blocks are always non-negative (i.e., $R_i^{(\omega)} \geq 0, \forall i \in \mathcal{N}, \forall \omega \in \Omega$). Constraint (12) basically says that the sum of the probabilities of the scenarios under which the link demand may not be satisfied (i.e., $\omega : u^{(\omega)} = 1$) is less than or equal to $1 - \beta$, which gives the same meaning as the chance constraint (2). We refer to the DEP of CSSP (after replacing (2) by (11) and (12)) by CSSP-DEP. We remark that CSSP-DEP is a binary integer linear program (BILP). The complexity of solving CSSP-DEP increases with the number of scenarios. Constraint (11) is repeated for each scenario $\omega \in \Omega$. $|\Omega|$ increases exponentially with the number of idle frequency blocks (i.e., N). For example, if we assume that each idle frequency block can support one of eight rates (as in IEEE 802.11a) according to a certain probability distribution, then $|\Omega| = 8^N$. To reduce the complexity of CSSP, in the following we propose a simplified version of CSSP.

b) *CSSP-modified*: Based on Markov’s inequality [23], the left-hand-side of the chance constraint (2) can be bounded from above as follows:

$$\Pr \left\{ \sum_{i=1}^N \tilde{R}_i x_i \geq d \right\} \leq \frac{\mathbb{E} \left[\sum_{i=1}^N \tilde{R}_i x_i \right]}{d} = \frac{\sum_{i=1}^N \mu_i x_i}{d}. \quad (13)$$

Hence, if constraint (2) is satisfied, the following inequality is also satisfied:

$$\sum_{i=1}^N \mu_i x_i \geq d\beta. \quad (14)$$

However, if (14) is satisfied, this does not necessarily imply that (2) is satisfied. Therefore, we make (14) more stringent by multiplying the right-hand-side of (14) with a constant $\kappa > 1$, so that constraint (14) becomes:

$$\sum_{i=1}^N \mu_i x_i \geq \kappa d\beta. \quad (15)$$

The value of κ is determined empirically such that (15) best approximates the chance constraint (2). If the chance constraint (2) is replaced with (15), CSSP becomes a standard *deterministic* SSP. In this SSP, the value of the i th frequency block is μ_i and the demand is $\kappa d\beta$. Although SSP is still an NP-hard problem, there exist a pseudo-polynomial ϵ -approximate algorithm and a polynomial-time greedy heuristic for solving it. Table I lists the complexity of various SSP solutions. Our procedure for solving the modified version of CSSP is summarized in Algorithm 1.

Algorithm 1 CSSP-modified

- 1: **Input:** $N, d, \beta, \kappa, \mu_i, i \in \mathcal{N}, p^{(\omega)}, \omega \in \Omega$
 - 2: **Output:** $x_i, i \in \mathcal{N}$, or declare the problem as infeasible
 - 3: Replace (2) in CSSP with (15), and solve the resulting (deterministic) SSP
 - 4: Evaluate (2) with respect to the obtained solution
 - 5: **If** (2) is satisfied **Then**
 - 6: Terminate, and report $x_i, i \in \mathcal{N}$
 - 7: **Else**
 - 8: Select more blocks from the unassigned ones, starting with the one that has the smallest expected rate until a solution to (2) is found. Then, terminate
 - 9: **End If**
 - 10: **If** assigning all blocks does not lead to a solution **Then**
 - 11: Declare the problem as infeasible
 - 12: **End If**
-

Constraint (2) is evaluated for a given block assignment as follows:

1. Sort the scenarios in a descending order according to their probabilities ($p^{(\omega)}$ for scenario $\omega \in \Omega$).
2. Starting from the most probable scenario, do the following:
 - Evaluate the link demand constraint $\sum_{i=1}^N R_i^{(\omega)} x_i \geq d$ under scenario $\omega \in \Omega$.
 - If scenario ω satisfies the link demand constraint, increment the probability of satisfying the link demand by $p^{(\omega)}$.
 - If the total probability of satisfying the link demand reaches or exceeds β , then the given block assignment satisfies (2). Otherwise, go to the next most probable scenario.
3. If all the scenarios have been considered and the total probability of satisfying the link demand is less than β , then the given block assignment does not satisfy (2).

It is to be mentioned that κ is a critical parameter of CSSP-modified. If κ is set to a very large value, then (15) might not be satisfied. On the other hand, if κ is set to a very small value, then the solution of the deterministic SSP might not be a feasible solution to CSSP. The value of κ that makes (15) best approximates (2) depends on the other system parameters, such as the number of data blocks, d , and β .

2) Adaptive Two-Stage Assignment:

a) *DEP*: To obtain the DEP of CSSPR, we first reformulate the first-stage chance constraint (5), as in CSSP. Then, the second-stage objective function (7) is substituted in (4). Then, the DEP of the CSSPR is given by:

Problem CSSPR-DEP:

$$\left\{ \begin{array}{l} \text{minimize} \\ x_i, y_i^{(\omega)}, u^{(\omega)} \\ i \in \mathcal{N}, \omega \in \Omega \end{array} \right\} \left\{ \sum_{i=1}^N \mu_i x_i - \sum_{\omega \in \Omega} p^{(\omega)} \left(\sum_{i=1}^N \alpha_i R_i^{(\omega)} y_i^{(\omega)} \right) \right\} \quad (16)$$

subject to (11)–(12)

$$y_i^{(\omega)} \leq x_i, \forall i \in \mathcal{N}, \forall \omega \in \Omega \quad (17)$$

$$\sum_{k=1}^N \left(x_k - y_k^{(\omega)} \right) R_k^{(\omega)} \geq d y_i^{(\omega)}, \forall i \in \mathcal{N}, \forall \omega \in \Omega \quad (18)$$

$$x_i, y_i^{(\omega)}, u^{(\omega)} \in \{0, 1\}, \forall i \in \mathcal{N}, \forall \omega \in \Omega. \quad (19)$$

The second term in the objective function (16) is the expected value of the objective function of the second-stage problem of CSSPR (i.e., (7)). Again, CSSPR-DEP is a BILP.

b) *CSSPR-modified*: The simplified version of CSSPR is summarized in Algorithm 2. In line 3 of Algorithm 2, we obtain a feasible first-stage solution to CSSPR from CSSP-modified (Algorithm 1). A feasible solution to CSSP is always feasible to CSSPR, because, as we mentioned earlier, CSSPR has a relatively complete recourse.

In line 9 of Algorithm 2, for a fixed first-stage solution $x_i, i \in \mathcal{N}$, and a specific scenario $\omega \in \Omega$, the second-stage problem reduces to a *deterministic* SSP, where only the blocks that have been assigned in the first stage are considered in this SSP. This can be seen as follows:

- Constraint (8) reduces to $y_i \leq 1, \forall i \in \mathcal{N}$, which is redundant, given constraint (10).
- Constraint (9) can be rearranged as:

$$\sum_{\substack{k=1 \\ k \neq i}}^N R_k^{(\omega)} y_k + \left(R_i^{(\omega)} + d \right) y_i \leq \sum_{k=1}^N R_k^{(\omega)} x_k. \quad (20)$$

Note that the right-hand-side of (20) is constant.

Algorithm 2 CSSPR-modified

- 1: **Input:** $N, d, \beta, \kappa, \mu_i, p^{(\omega)}, \omega \in \Omega, R_i^{(\omega)}, i \in \mathcal{N}, \omega \in \Omega$
 - 2: **Output:** $x_i, y_i^{(\omega)}, i \in \mathcal{N}, \omega \in \Omega$, or declare the problem as infeasible
 - 3: Use Algorithm 1 to obtain a feasible $x_i, i \in \mathcal{N}$
 - 4: **If** there is no feasible $x_i, i \in \mathcal{N}$ **Then**
 - 5: Declare the problem as infeasible
 - 6: **End If**
 - 7: **For** each $\omega \in \Omega$ **Do**
 - 8: Fix $x_i, i \in \mathcal{N}$, as obtained from Algorithm 1
 - 9: Solve the second-stage problem for scenario ω , to determine the blocks to be excluded from the initial assignment
 - 10: **End For**
-

IV. BATCH ASSIGNMENT

A. Problem Formulation

Assigning channels to multiple links in a wireless network can be done sequentially, i.e., one link at a time. In this case, the “exact”⁴ and the modified assignment schemes proposed in Section III-B can be applied sequentially (note that the spectrum status needs to be updated after each link assignment, before executing the next link assignment). The links can be considered in different orders. In Section V, we implement

⁴By exact schemes, we mean the solutions of the CSSP-DEP and CSSPR-DEP optimization problems.

two ordering approaches: (i) assign channels to links in an ascending order of link demands (denoted by SEQ_{asc}), and (ii) assign channels in a descending order of link demands (denoted by SEQ_{dsc}).

However, sequential assignment (in general) does not achieve the optimal network-wide spectrum efficiency. Another way of assigning channels to multiple links is the *batch assignment*, in which all links are considered simultaneously. The batch assignment approach can be implemented in a centralized or a distributed manner. To implement the batch assignment in a distributed network, one may apply the *access window (AW)* concept used in [24] and [25], where each link broadcasts its rate demand in a given slot. Each link waits for a certain amount of time to collect the demands of other links in the network before executing the joint assignment problem.

We propose two batch assignment schemes: A static single-stage and an adaptive two-stage assignment schemes.

1) *Static Single-Stage Assignment*: In here, we extend the single-link static assignment scheme proposed in Section III-A1 to multiple links. Specifically, instead of a single chance constraint, we add a chance constraint for each link. Moreover, instead of the single-link expected rate, the objective function becomes the network-wide expected rate. Let $x_{ij}, i \in \mathcal{N}$ and $j \in \mathcal{L}$, be a binary variable; $x_{ij} = 1$ if block i is assigned to link j , and zero otherwise. Then, the static batch assignment problem is formulated as a chance-constrained stochastic multiple SSP (CMSSP) as follows:

Problem CMSSP:

$$\underset{\{x_{ij}, i \in \mathcal{N}, j \in \mathcal{L}\}}{\text{minimize}} \sum_{j=1}^L \sum_{i=1}^N \mu_i x_{ij} \quad (21)$$

$$\text{subject to } \Pr \left\{ \sum_{i=1}^N \tilde{R}_i x_{ij} \geq d_j \right\} \geq \beta_j, \forall j \in \mathcal{L} \quad (22)$$

$$\sum_{j=1}^L x_{ij} \leq 1, \forall i \in \mathcal{N} \quad (23)$$

$$x_{ij} \in \{0, 1\}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L} \quad (24)$$

where constraint (23) ensures that an idle block can be allocated to one link only.

Remark 1: In the above CMSSP formulation, links are assumed to interfere with each other. Therefore, constraint (23) ensures that an idle block cannot be assigned to more than one link at the same time. Our CMSSP formulation can be extended to accommodate scenarios where links do not necessarily conflict with each other. To do this, for each pair of links k and $m \in \mathcal{L}$, we introduce a binary variable y_{km} . $y_{km} = 1$ if links k and m are conflicting, and $y_{km} = 0$ otherwise. Then, constraint (23) is replaced by the following constraint:

$$x_{ik} + x_{im} \leq 2 - y_{km}, \forall i \in \mathcal{N}, \forall k, m \in \mathcal{L}. \quad (25)$$

2) *Adaptive Two-Stage Assignment*: The adaptive batch assignment scheme is formulated as a two-stage CMSSP with recourse (CMSSPR), which consists of two stages. The first stage is similar to CMSSP. The second stage of CMSSPR is more complicated than the second stage of CSSPR. In the second stage of CSSPR, blocks can only be removed (if they

are excess). In the second-stage of CMSSPR, excess blocks can be removed from over-satisfied links, and also they can be added to under-satisfied links, which makes the formulation more involved.

In addition to maximizing the discounted sum-rate of the blocks that can be taken from over-satisfied links (as in CSSPR), the second stage of CMSSPR aims to: (i) maximize the discounted sum-rate of the blocks that can be added to the under-satisfied links, and (ii) minimize the sum (over all links) of the difference between the link demand and its assigned rate. Let $x_{ij}, i \in \mathcal{N}, j \in \mathcal{L}$, be defined as in CMSSP. Let $y_{ij}, i \in \mathcal{N}, j \in \mathcal{L}$, be a binary variable; $y_{ij} = 1$ if block i is removed from link j , and zero otherwise. Also, let $z_{ij}, i \in \mathcal{N}, j \in \mathcal{L}$, be a binary variable; $z_{ij} = 1$ if block i is added to link j , and zero otherwise. Then, the objective function of the second stage of CMSSPR can be expressed as follows:

$$\underset{\{y_{ij}, z_{ij}, i \in \mathcal{N}, j \in \mathcal{L}\}}{\text{minimize}} \left\{ - \sum_{j=1}^L \sum_{i=1}^N \alpha_i \tilde{R}_i (y_{ij} + z_{ij}) + \sum_{j=1}^L \left| d_j - \sum_{i=1}^N (x_{ij} - y_{ij} + z_{ij}) \tilde{R}_i \right| \right\} \quad (26)$$

where α_i is defined as in CSSPR. The term $\sum_{i=1}^N (x_{ij} - y_{ij} + z_{ij}) \tilde{R}_i$ represents the total rate assigned to link j . We can remove the absolute value and simplify (26) by introducing an auxiliary variable for each link. Let $e_j, j \in \mathcal{L}$, be the auxiliary variables. Then (26) can be replaced by:

$$\underset{\{y_{ij}, z_{ij}, e_j, i \in \mathcal{N}, j \in \mathcal{L}\}}{\text{minimize}} \left\{ - \sum_{j=1}^L \sum_{i=1}^N \alpha_i \tilde{R}_i (y_{ij} + z_{ij}) + \sum_{j=1}^L e_j \right\} \quad (27)$$

$$\text{subject to } e_j \geq d_j - \sum_{i=1}^N (x_{kj} - y_{kj} + z_{kj}) \tilde{R}_k, \forall j \in \mathcal{L} \quad (28)$$

$$e_j \geq \sum_{i=1}^N (x_{kj} - y_{kj} + z_{kj}) \tilde{R}_k - d_j, \forall j \in \mathcal{L}. \quad (29)$$

The second-stage problem of CMSSPR includes the following constraints:

1. A block can be released only if it has been already assigned.
2. A block can be taken only from over-satisfied links.
3. A block can be assigned to under-satisfied links only.
4. A block can be assigned to an under-satisfied link only if it can be released from an over-satisfied link.
5. A released block from an over-satisfied link cannot be reassigned to the same link.
6. A released block can be assigned only to one under-satisfied link.

The first two constraints are the same as in CSSPR, and can be enforced by adding:

$$y_{ij} \leq x_{ij}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L} \quad (30)$$

$$\sum_{k=1}^N (x_{kj} - y_{kj}) \tilde{R}_k \geq d_j y_{ij}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}. \quad (31)$$

Constraint 3 above can be ensured by adding:

$$\sum_{k=1}^N (x_{kj} - y_{kj} + z_{kj}) \tilde{R}_k \geq d_j \eta_{ij} + z_{ij} \tilde{R}_i, \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{L} \quad (32)$$

$$z_{ij} \leq 1 - \eta_{ij}, \eta_{ij} \in \{0, 1\}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L} \quad (33)$$

Note that if the link demand is already met, η_{ij} will be one which enforces z_{ij} to be zero. On the other hand, if the link demand is not satisfied, η_{ij} will be zero, and in this case z_{ij} can be zero or one. Constraint 4 can be met by imposing:

$$z_{ij} \leq \sum_{k=1}^L y_{ik}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}. \quad (34)$$

Finally, to ensure satisfying constraints 5 and 6, we add the following two constraints, respectively:

$$z_{ij} \leq 1 - y_{ij}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L} \quad (35)$$

$$\sum_{j=1}^L z_{ij} \leq 1, \forall i \in \mathcal{N}. \quad (36)$$

The resulting CMSSPR formulation of the adaptive batch assignment is given by:

Problem CMSSPR:

$$\begin{aligned} & \underset{\{x_{ij}, i \in \mathcal{N}, j \in \mathcal{L}\}}{\text{minimize}} \left\{ \sum_{j=1}^L \sum_{i=1}^N \mu_i x_{ij} + \mathbb{E} \left[h(\mathbf{x}, \tilde{\mathbf{R}}) \right] \right\} \\ & \text{subject to} \quad (22) \text{--}(24) \end{aligned} \quad (37)$$

where $h(\mathbf{x}, \tilde{\mathbf{R}})$ is the optimal value of the second-stage problem, which is given by:

$$\begin{aligned} & \underset{\left\{ \begin{array}{l} y_{ij}, z_{ij} \\ \eta_{ij}, e_j \\ i \in \mathcal{N}, j \in \mathcal{L} \end{array} \right\}}{\text{minimize}} \left\{ - \sum_{j=1}^L \sum_{i=1}^N \alpha_i \tilde{R}_i (y_{ij} + z_{ij}) + \sum_{j=1}^L e_j \right\} \\ & \text{subject to} \quad (28) \text{--}(36). \end{aligned} \quad (38)$$

Remark 2: In our CMSSPR formulation, all extra blocks are removed from over-satisfied links, even if they will not be used by other links during the current AW, i.e., we can have a situation where $y_{ij} = 1$ and $z_{ij'} = 0, \forall j' \neq j$. The reason is that new demands may arrive during the next AW, which will use the excess blocks from the previous AW.

B. Problem Reformulation and Solution Approach

Following the same techniques used in Section III-B, we derive in this section the DEPs of CMSSP and CMSSPR.

1) *Static Single-Stage Assignment:* The DEP of CMSSP is given by:

Problem CMSSP-DEP:

$$\begin{aligned} & \underset{\left\{ \begin{array}{l} x_{ij}, u_j^{(\omega)} \\ i \in \mathcal{N}, j \in \mathcal{L}, \omega \in \Omega \end{array} \right\}}{\text{minimize}} \sum_{j=1}^L \sum_{i=1}^N \mu_i x_{ij} \\ & \text{subject to} \quad \sum_{i=1}^N R_i^{(\omega)} x_{ij} \geq d_j \left(1 - u_j^{(\omega)} \right), \forall j \in \mathcal{L}, \forall \omega \in \Omega \end{aligned} \quad (39)$$

(40)

TABLE II
RATE DISTRIBUTION OF FIVE FREQUENCY BLOCKS USED IN THE SINGLE-LINK ASSIGNMENT SCHEMES ($r_1 = 0$ Mbps, $r_2 = 1$ Mbps, $r_3 = 2$ Mbps, $r_4 = 4$ Mbps, AND $r_5 = 6$ Mbps)

Idle block	Pr[r_1]	Pr[r_2]	Pr[r_3]	Pr[r_4]	Pr[r_5]
IB ₁	0.1	0.8	0.1	0.0	0.0
IB ₂	0.05	0.1	0.7	0.1	0.05
IB ₃	0.0	0.05	0.4	0.5	0.05
IB ₄	0.0	0.05	0.1	0.8	0.05
IB ₅	0.0	0.0	0.1	0.4	0.5

$$\sum_{\omega \in \Omega} p^{(\omega)} u_j^{(\omega)} \leq 1 - \beta_j, \forall j \in \mathcal{L} \quad (41)$$

$$\sum_{j=1}^L x_{ij} \leq 1, \forall i \in \mathcal{N} \quad (42)$$

$$x_{ij}, u_j^{(\omega)} \in \{0, 1\}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}, \forall \omega \in \Omega. \quad (43)$$

2) *Adaptive Two-Stage Assignment:* The DEP of CMSSPR is given by (44)–(52), shown at the bottom of the next page.

To reduce the complexity of CMSSP and CMSSPR, links may be considered sequentially according to a certain order. For each link assignment, CSSP-modified and CSSPR-modified can be used for the static and adaptive models, respectively. We refer to the resulting schemes as CMSSP-modified and CMSSPR-modified, respectively.

V. NUMERICAL EVALUATION

In this section, we numerically evaluate the performance of our proposed assignment schemes. All schemes are implemented in CPLEX.

A. Single-Link Assignment

In this part, we evaluate our proposed single-link assignment schemes (CSSP, CSSP-modified, CSSPR, and CSSPR-modified). We compare CSSP with CSSPR. Furthermore, we study the performance of the modified CSSP and CSSPR schemes relative to the optimal CSSP and CSSPR schemes.

We set $N = 5$. Each frequency block can support one of the following rates: $r_1 = 0$ Mbps,⁵ $r_2 = 1$ Mbps, $r_3 = 2$ Mbps, $r_4 = 4$ Mbps, and $r_5 = 6$ Mbps, with the probability distribution shown in Table II. α_i is set to 0.8 for all $i \in \mathcal{N}$. We solve CSSP and CSSPR for different combinations of β and d values. Our proposed CSSP-modified and CSSPR-modified algorithms produce upper bounds for both models. We set $\kappa = 1.5$ in (15), as it is found to be efficient for most instances.

Fig. 2 depicts the optimal expected link throughput of CSSP and CSSPR versus β for $d = 6$ and 10 Mbps. The figure shows that for $\beta \geq 0.7$, the optimal expected link throughput of both CSSP and CSSPR generally exceeds the demand. Moreover, Fig. 2 shows that the expected link throughput of CSSP increases with β . Increasing β makes the chance constraint in (2) more stringent, resulting in over-satisfying the link demand. In

⁵This corresponds to the case when the block is occupied by PUs.

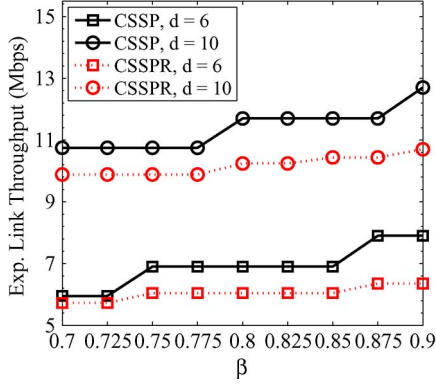


Fig. 2. Expected link throughput vs. β for CSSP and CSSPR (single-link assignment).

contrast, the expected throughput of CSSPR does not increase significantly with β . This is due to the recourse action in the second stage of CSSPR, in which we attempt to remove additional blocks. This results in a smaller net link throughput, leaving more channels to the links that will be subsequently assigned. This leads to increasing the admission rate of the sequential assignment schemes.

Fig. 3 shows the expected link throughput of CSSP and CSSP-modified vs. d for $\beta = 0.7, 0.8$, and 0.9 . Similarly, Fig. 4 shows the expected link throughput of CSSPR and CSSPR-modified. As can be seen, for large values of β and d both CSSP and CSSPR are infeasible. Also, these figures show that CSSP-modified and CSSPR-modified achieve a relatively close-to-optimal performance, and at some instances the

solutions of CSSP and CSSP-modified coincide (similarly for CSSPR and CSSPR-modified).

B. Multi-Link Assignment

In this part, we consider multiple links. We study both the sequential as well as the batch assignment schemes. We set $N = 8$. Each frequency block can support one of the following rates: $r_1 = 0$ Mbps, $r_2 = 1$ Mbps, and $r_3 = 4$ Mbps, with the probability distribution shown in Table V. $\alpha_i = 0.8, \forall i \in \mathcal{N}$.

1) *Sequential Assignment*: We implement two sequential assignment schemes, SEQ_{asc} and SEQ_{dsc} . In SEQ_{asc} , links are ordered in an ascending order of their rate demands, then considered one at a time. In contrast, links are ordered in a descending order of their demands in the SEQ_{dsc} scheme. To compare SEQ_{asc} and SEQ_{dsc} , we use the CSSP scheme in assigning each link. We set $L = 4$ and run SEQ_{asc} and SEQ_{dsc} for different values of β ($\beta_j = \beta, \forall j \in \mathcal{L}$). The rate demands of the four links are: $d_1 = 6$ Mbps, $d_2 = 4$ Mbps, $d_3 = 2.5$ Mbps, and $d_4 = 1.5$ Mbps. We evaluate the percentage of admitted links (i.e., the number of satisfied links divided by L) for different values of β . Fig. 5 shows the percentage of admitted links vs. β . As shown in the figure, the order of considering links matters, and the best way of ordering links depends on the value of β . When β equals 0.7 or 0.75 , if links are considered in a descending order of their demands then all of them will be admitted. However, if they are considered in an ascending order of their demands, only three links out of four will be admitted. On the other hand, when β equals 0.85 or 0.9 , only 50% of the links are admitted in SEQ_{dsc} compared to 75% of the links in SEQ_{asc} .

Problem CMSSPR-DEP:

$$\begin{aligned} & \underset{\{x_{ij}, y_{ij}^{(\omega)}, z_{ij}^{(\omega)}, u_j^{(\omega)}, e_j^{(\omega)}, \eta_{ij}^{(\omega)}\}}{\text{minimize}} \left\{ \sum_{j=1}^L \sum_{i=1}^N \mu_i x_{ij} - \sum_{\omega \in \Omega} p^{(\omega)} \left(\sum_{j=1}^L \sum_{i=1}^N \alpha_i R_i^{(\omega)} \left(y_{ij}^{(\omega)} + z_{ij}^{(\omega)} \right) - \sum_{j=1}^L e_j^{(\omega)} \right) \right\} \\ & \text{subject to (40)–(42)} \end{aligned} \quad (44)$$

$$e_j^{(\omega)} \geq d_j - \sum_{k=1}^N \left(x_{kj} - y_{kj}^{(\omega)} + z_{kj}^{(\omega)} \right) R_k^{(\omega)}, \forall j \in \mathcal{L}, \forall \omega \in \Omega \quad (45)$$

$$e_j^{(\omega)} \geq \sum_{k=1}^N \left(x_{kj} - y_{kj}^{(\omega)} + z_{kj}^{(\omega)} \right) R_k^{(\omega)} - d_j, \forall j \in \mathcal{L}, \forall \omega \in \Omega \quad (46)$$

$$y_{ij}^{(\omega)} \leq x_{ij}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}, \forall \omega \in \Omega \quad (47)$$

$$\sum_{k=1}^N \left(x_{kj} - y_{kj}^{(\omega)} \right) R_k^{(\omega)} \geq d_j y_{ij}^{(\omega)}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}, \forall \omega \in \Omega \quad (48)$$

$$\sum_{k=1}^N \left(x_{kj} - y_{kj}^{(\omega)} + z_{kj}^{(\omega)} \right) R_k^{(\omega)} \geq d_j \eta_{ij}^{(\omega)} + z_{ij}^{(\omega)} R_i^{(\omega)}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}, \forall \omega \in \Omega \quad (49)$$

$$z_{ij}^{(\omega)} \leq 1 - \eta_{ij}^{(\omega)}, z_{ij}^{(\omega)} \leq \sum_{k=1}^L y_{ik}^{(\omega)}, z_{ij}^{(\omega)} \leq 1 - y_{ij}^{(\omega)}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}, \forall \omega \in \Omega \quad (50)$$

$$\sum_{j=1}^L z_{ij}^{(\omega)} \leq 1, \forall i \in \mathcal{N}, \forall \omega \in \Omega \quad (51)$$

$$x_{ij}, u_j^{(\omega)}, y_{ij}^{(\omega)}, z_{ij}^{(\omega)}, \eta_{ij}^{(\omega)} \in \{0, 1\}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}, \forall \omega \in \Omega. \quad (52)$$

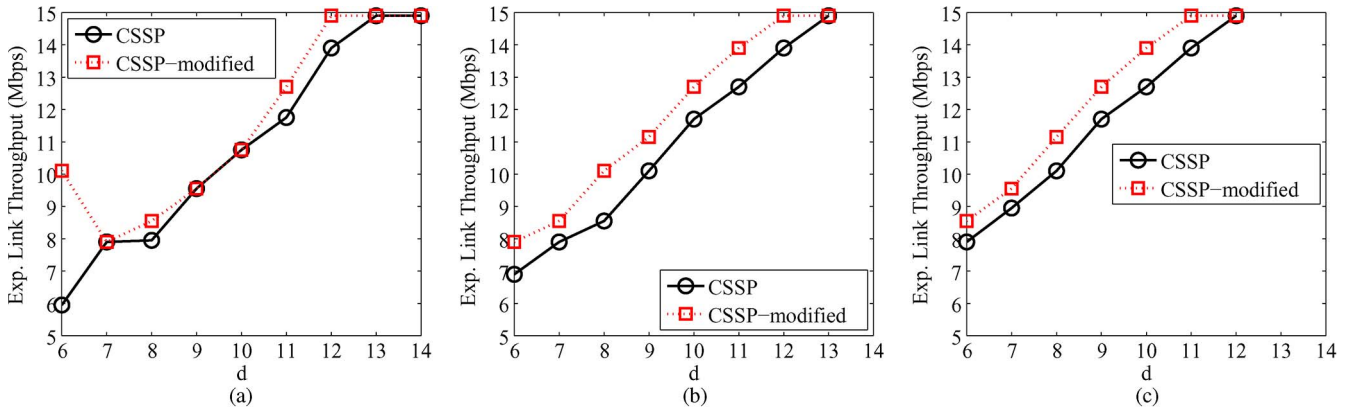


Fig. 3. Expected link throughput vs. d for CSSP (single-link assignment). (a) $\beta = 0.7$. (b) $\beta = 0.8$. (c) $\beta = 0.9$.

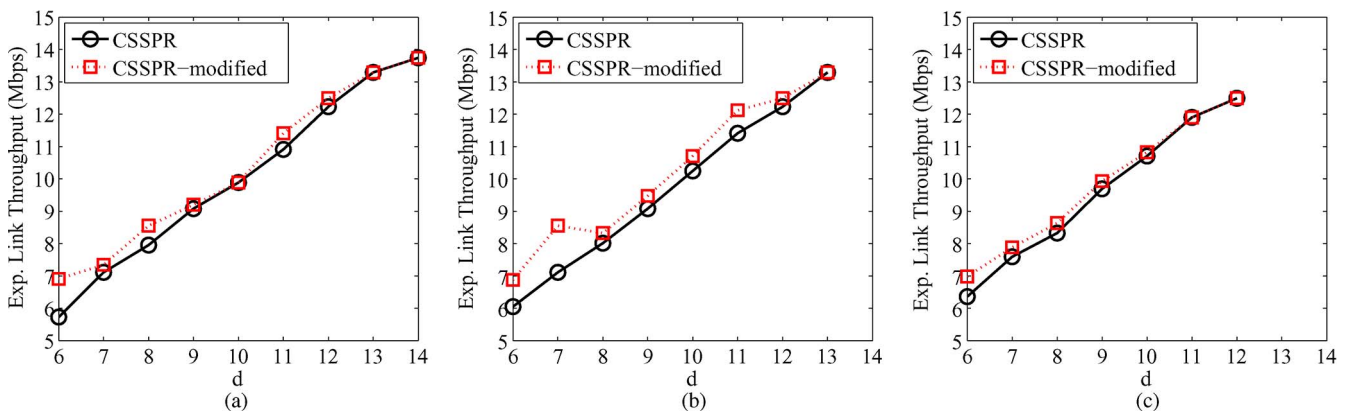


Fig. 4. Expected link throughput vs. d for CSSPR (single-link assignment). (a) $\beta = 0.7$. (b) $\beta = 0.8$. (c) $\beta = 0.9$.

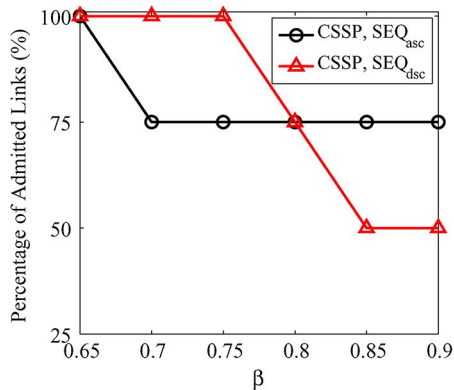


Fig. 5. Percentage of admitted links for sequential (CSSP-based) assignment schemes.

Because in the batch assignment all links are considered simultaneously, it achieves the best performance (i.e., in terms of the percentage of admitted links) of all sequential assignment schemes. In here, we implemented two sequential assignment schemes (SEQ_{asc} and SEQ_{dsc}) to illustrate the fact that the order of considering links is important. Moreover, the best order of considering links depends on β and the link demands. The batch assignment achieves an admission rate of 100% (if the available resources are sufficient to probabilistically satisfy the link demands), which may not be achieved by any sequential scheme for all values of β . This is because a sequential scheme

TABLE III
EXPECTED THROUGHPUT (IN Mbps) OF THE SINGLE-STAGE MULTI-LINK ASSIGNMENT SCHEMES

$(\beta_1, \beta_2, \beta_3)$	(d_1, d_2, d_3)	Exp. Throughput (CMSSP)	Exp. Throughput (CMSSP-modified)
(0.7, 0.7, 0.7)	(7, 13, 14)	37.7	37.7
(0.8, 0.8, 0.8)	(7, 13, 13)	36.7	37.7
(0.9, 0.9, 0.9)	(8, 11, 12)	38.9	40.95

that is optimal for a given value of β may not be optimal for other values of β .

2) *Batch Assignment*: To show the advantage of the adaptive model (CMSSPR) compared to the static model (CMSSP), we set $L = 4$ and run CMSSP and CMSSPR for different values of β . The rate demands of the four links are: $d_1 = 2$ Mbps, $d_2 = 1$ Mbps, $d_3 = 1.5$ Mbps, and $d_4 = 1$ Mbps. We evaluate the probability of demand unsatisfaction (i.e., the sum of the probabilities of the scenarios under which the rate demand is not satisfied) of both CMSSP and CMSSPR for different values of β . Fig. 6 shows the probability of demand unsatisfaction for the first and third links as a function of β . The added second-stage in CMSSPR reduces the probability of demand unsatisfaction to almost zero. It redistributes the blocks between the links, trying to satisfy all demands. Note that constraint (22)

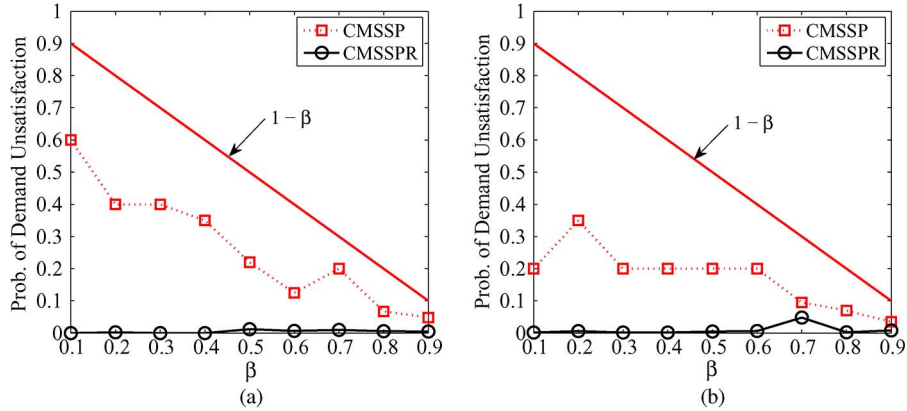


Fig. 6. Probability of demand unsatisfaction vs. β for links 1 and 3. (a) Link # 1 (demand = 2 Mbps). (b) Link # 3 (demand = 1.5 Mbps).

TABLE IV
EXPECTED THROUGHPUT (IN Mbps) OF THE TWO-STAGE
MULTI-LINK ASSIGNMENT SCHEMES

$(\beta_1, \beta_2, \beta_3)$	(d_1, d_2, d_3)	Exp. Throughput (CMSSPR)	Exp. Throughput (CMSSPR-modified)
(0.7, 0.7, 0.7)	(7, 13, 14)	34.1392	34.374
(0.8, 0.8, 0.8)	(7, 13, 13)	33.6872	35.13
(0.9, 0.9, 0.9)	(8, 11, 12)	32.71212	33.01922

TABLE V
RATE DISTRIBUTION OF EIGHT FREQUENCY BLOCKS USED
IN THE MULTI-LINK ASSIGNMENT SCHEMES

Idle block	$\Pr[r_1 = 0 \text{ Mbps}]$	$\Pr[r_2 = 1 \text{ Mbps}]$	$\Pr[r_3 = 4 \text{ Mbps}]$
IB ₁	0.1	0.3	0.6
IB ₂	0.2	0.4	0.4
IB ₃	0.0	0.7	0.3
IB ₄	0.05	0.15	0.8
IB ₅	0.25	0.35	0.4
IB ₆	0.05	0.75	0.2
IB ₇	0.0	0.5	0.5
IB ₈	0.15	0.2	0.65

enforces the probability of demand unsatisfaction of CMSSP to be smaller than $1 - \beta$. As shown in the figure, the probability of demand unsatisfaction reduces with β . Furthermore, this probability is higher for link 1 than link 3 because d_1 is greater than d_3 .

Finally, in Table III we compare the performance of CMSSP with CMSSP-modified. Similarly, in Table IV we compare the performance of CMSSPR with CMSSPR-modified. In Tables III and IV, we consider three links and 15 frequency blocks. The frequency blocks are classified into five categories with each category containing three blocks that have the same rate distribution (shown in Table II). In CMSSP-modified, the CSSP-modified scheme is applied to the three links sequentially in a descending order of their rate demands. As shown in Table III, the performance gap between CMSSP and CMSSP-modified varies with the values of β_j and d_j , $j \in \mathcal{L}$. In some cases, CMSSP and CMSSP-modified have the same performance. In CMSSPR-modified, the CSSPR-modified scheme is applied to the links sequentially in a descending order of their rate demands. Again, the performance gap between CMSSPR and CMSSPR-modified varies with the values of β_j and d_j , $j \in \mathcal{L}$.

VI. CONCLUSION AND FUTURE RESEARCH

In this paper, we studied the problem of performing GBA channel assignment with bonding and aggregation under channel-quality uncertainty. We considered both sequential as well as batch assignment schemes. For each scheme, we developed two assignment models: a static single-stage and an adaptive two-stage. All assignment schemes were formulated using

stochastic optimization techniques. Optimal solutions for the formulated assignment problems were obtained. Furthermore, we designed computationally efficient simplified stochastic assignment algorithms. We conducted numerical experiments to: (i) examine the efficiency of our simplified assignment schemes under different values of the model parameters, (ii) compare the performance of the static assignment with the adaptive assignment, and (iii) compare the sequential assignment schemes with the batch schemes. Our results indicate the following. First, the performance of the simplified assignment schemes is very close to the optimal, especially in the adaptive assignment. Second, the adaptive assignment is more powerful than the static assignment in uncertain environments. In particular, the probability of demand unsatisfaction of the adaptive assignment scheme is close to zero, whereas this probability can reach to 0.6 in the static scheme (depending on the system parameters). Third, the batch assignment achieves the best admission rate of all sequential assignment schemes. In our experiments (for four links), the admission rate of the batch scheme can reach up to 25% higher than that of any of the two sequential schemes, SEQ_{asc} and SEQ_{dsc} .

As future research, we aim to investigate the problem of designing efficient ϵ -approximate algorithms for solving our proposed stochastic assignment problems. As a first step, we plan to develop an ϵ -approximate algorithm for solving the CSSP formulation. To achieve this, we propose to start with the ϵ -approximate algorithm of the deterministic SSP [26]. Consider a particular combination of idle frequency blocks. Instead of computing the total rate that these blocks can support, as in the ϵ -approximate algorithm of the deterministic SSP, we need

to compute the maximum rate that these blocks can support with probability greater than β . Such computation requires considering all rate scenarios that can happen and their probability of occurrence (the number of scenarios is exponential in the number of idle blocks). To address this computation complexity, we propose to design a ‘Trim’ function similar to the one used in the ϵ -approximate algorithm of the deterministic SSP [26] such that a large portion of the scenarios can be neglected. The trimming criterion will depend on the difference between the rates supported under various scenarios as well as the probabilities of these scenarios. Such line of investigation will be further explored in our future work.

ACKNOWLEDGMENT

The authors would like to thank Dr. Güzin Bayraksan (Ohio State University) and Dr. Fujun Lan (University of Maryland School of Medicine) for their helpful discussions on stochastic optimization.

REFERENCES

- [1] M. J. Abdel-Rahman, F. Lan, and M. Krunz, “Spectrum-efficient stochastic channel assignment for opportunistic networks,” in *Proc. IEEE GLOBECOM Conf.*, Dec. 2013, pp. 1272–1277.
- [2] FCC, “Second memorandum opinion and order in the matter of unlicensed operation in the TV broadcast bands (ET Docket No. 04-186),” Additional Spectrum for Unlicensed Devices Below 900 MHz and in 3 GHz Band, Washington, DC, USA, (EC Docket No. 02-380), Sep. 23, 2010.
- [3] K. Harrison, S. M. Mishra, and A. Sahai, “How much white-space capacity is there?” in *Proc. IEEE DySPAN Conf.*, 2010, pp. 1–10.
- [4] Y. Wu, E. Pliszka, B. Caron, P. Bouchard, and G. Chouinard, “Comparison of terrestrial DTV transmission systems: The ATSC 8-VSB, the DVB-T COFDM, and the ISDB-T BST-OFDM,” *IEEE Trans. Broadcast.*, vol. 46, no. 2, pp. 101–113, Jun. 2000.
- [5] V. Angelakis, S. Papadakis, V. Siris, and A. Traganitis, “Adjacent channel interference in 802.11a is harmful: Testbed validation of a simple quantification model,” *IEEE Commun. Mag.*, vol. 49, no. 3, pp. 160–166, Mar. 2011.
- [6] A. Zubow and R. Sombrutski, “Adjacent channel interference in IEEE 802.11n,” in *Proc. IEEE WCNC Conf.*, Apr. 2012, pp. 1163–1168.
- [7] D. Skordoulis et al., “IEEE 802.11n MAC frame aggregation mechanisms for next-generation high-throughput WLANs,” *IEEE Wireless Commun. Mag.*, vol. 15, no. 1, pp. 40–47, Feb. 2008.
- [8] L. Deek, E. Garcia-Villegas, E. Belding, S.-J. Lee, and K. Almeroth, “The impact of channel bonding on 802.11n network management,” in *Proc. ACM CoNEXT Conference*, Dec. 2011, p. 11.
- [9] L. Deek, E. Garcia-Villegas, E. Belding, S. Lee, and K. Almeroth, “Intelligent channel bonding in 802.11n WLANs,” *IEEE Trans. Mobile Comput.*, vol. 13, no. 6, pp. 1242–1255, Jun. 2014.
- [10] M. Park, “IEEE 802.11ac: Dynamic bandwidth channel access,” in *Proc. IEEE ICC Conf.*, Jun. 2011, pp. 1–5.
- [11] O. Bejarano, E. Knightly, and M. Park, “IEEE 802.11ac: From channelization to multi-user MIMO,” *IEEE Commun. Mag.*, vol. 51, no. 10, pp. 84–90, Oct. 2013.
- [12] G. Yuan, X. Zhang, W. Wang, and Y. Yang, “Carrier aggregation for LTE-advanced mobile communication systems,” *IEEE Commun. Mag.*, vol. 48, no. 2, pp. 88–93, Feb. 2010.
- [13] H. Zhu and J. Wang, “Chunk-based resource allocation in OFDMA systems—Part I: Chunk allocation,” *IEEE Trans. Commun.*, vol. 57, no. 9, pp. 2734–2744, Sep. 2009.
- [14] H. Zhu and J. Wang, “Chunk-based resource allocation in OFDMA systems—Part II: Joint chunk, power and bit allocation,” *IEEE Trans. Commun.*, vol. 60, no. 2, pp. 499–509, Sep. 2012.
- [15] L. Lei, V. Angelakis, and D. Yuan, “Performance analysis of chunk-based resource allocation in wireless OFDMA systems,” in *Proc. IEEE Int. Workshop CAMAD Commun. Links Netw.*, 2012, pp. 90–94.
- [16] I. Macaluso, L. DaSilva, and L. Doyle, “Learning Nash equilibria in distributed channel selection for frequency-agile radios,” in *Proc. ECAI Workshop Artif. Intell. Telecommun. Sensor Netw.*, Aug. 2012, pp. 7–10.

- [17] Y. Xiao, T. Forde, I. Macaluso, L. DaSilva, and L. Doyle, “Spatial spectrum sharing-based carrier aggregation for heterogeneous networks,” in *Proc. IEEE GLOBECOM Conf.*, 2012, pp. 2591–2596.
- [18] Y. Xiao, C. Yuen, P. Di Francesco, and L. DaSilva, “Dynamic spectrum scheduling for carrier aggregation: A game theoretic approach,” in *Proc. IEEE ICC Conf.*, 2013, pp. 2672–2676.
- [19] G. S. Uyanik, M. J. Abdel-Rahman, and M. Krunz, “Optimal guard-band-aware channel assignment with bonding and aggregation in multi-channel systems,” in *Proc. IEEE GLOBECOM Conf.*, Dec. 2013, pp. 4898–4903.
- [20] G. S. Uyanik, M. J. Abdel-Rahman, and M. Krunz, “Optimal channel assignment with aggregation in multi-channel systems: A resilient approach to adjacent-channel interference,” *Ad Hoc Netw. J.*, vol. 20, pp. 64–76, Sep. 2014.
- [21] S. Martello and P. Toth, *Knapsack Problems: Algorithms and Computer Implementations*. Hoboken, NJ, USA: Wiley, 1990.
- [22] P. Kall and S. W. Wallace, *Stochastic Programming*. Hoboken, NJ, USA: Wiley, 1994.
- [23] A. Klappenecker, “Markov’s Inequality.” [Online]. Available: <http://faculty.cs.tamu.edu/klappi/csce689-s10/markov.pdf>.
- [24] A. Muqattash and M. Krunz, “POWMAC: A single-channel power-control protocol for throughput enhancement in wireless ad hoc networks,” *IEEE J. Sel. Areas Commun.*, vol. 23, no. 5, pp. 1067–1084, May 2005.
- [25] H. Bany Salameh and M. Krunz, “Adaptive power-controlled MAC protocols for improved throughput in hardware-constrained cognitive radio networks,” *Ad Hoc Netw. J.*, vol. 9, no. 7, pp. 1127–1139, Sep. 2011.
- [26] T. H. Cormen, R. L. Rivest, and C. E. Leiserson, *Introduction to Algorithms*, 2nd ed. New York, NY, USA: McGraw-Hill, 2001.



Mohammad J. Abdel-Rahman (S’12–M’15) received the Ph.D. degree in electrical and computer engineering from the University of Arizona in 2014. He is a Postdoctoral Associate in the Electrical and Computer Engineering Department at Virginia Polytechnic Institute and State University. His research interests include the areas of wireless communications and networking, with emphasis on dynamic spectrum access networks, wireless security, resource management, adaptive protocols, satellite communications, and wireless sensor networks. He serves as a reviewer for several international conferences and journals.



Marwan Krunz (S’93–M’95–SM’04–F’10) received the Ph.D. degree in electrical engineering from Michigan State University in 1995. He is a Professor of ECE and CS at the University of Arizona. He is the Site Codirector at the US National Science Foundation (NSF) Broadband Wireless Access and Applications Center. He joined the University of Arizona in January 1997, after a brief postdoctoral stint at the University of Maryland. In 2010, he was a Visiting Chair of Excellence at the University of Carlos III de Madrid. He previously held other visiting research positions at INRIA-Sophia Antipolis, HP Labs, University of Paris VI, University of Paris V, and US West Advanced Technologies. His research interests include the areas of wireless communications and networking, with emphasis on resource management, adaptive protocols, and security issues. He has published more than 215 journal articles and peer-reviewed conference papers, and is a coinventor on five US patents. He received the 2012 IEEE TCCC Outstanding Service Award. He received the US NSF CAREER Award in 1998. He currently serves on the editorial board for the IEEE TRANSACTIONS ON COGNITIVE COMMUNICATIONS AND NETWORKING. Previously, he served on the editorial boards for the IEEE/ACM TRANSACTIONS ON NETWORKING, IEEE TRANSACTIONS ON MOBILE COMPUTING, IEEE TRANSACTIONS ON NETWORK AND SERVICE MANAGEMENT, *Computer Communications Journal*, and the *IEEE Communications Interactive Magazine*. He was the General Cochair for WiSec’12, and served as a TPC Chair for INFOCOM’04, SECON’05, and WoWMoM’06. He was the keynote speaker, an invited panelist, and a tutorial presenter at numerous international conferences. He is an Arizona Engineering Faculty fellow (2011–2014), and a IEEE Communications Society Distinguished lecturer (2013 and 2014).

Stochastic Guard-Band-Aware Channel Assignment With Bonding and Aggregation for DSA Networks

Mohammad J. Abdel-Rahman, *Member, IEEE*, and Marwan Krunz, *Fellow, IEEE*

Abstract—Fading and shadowing along with the primary user dynamics make channel quality in dynamic spectrum access networks uncertain. Furthermore, the imperfect design of filters and amplifiers in wireless devices motivates the need for guard-bands (GBs) to prevent adjacent-channel interference. In this paper, we develop novel stochastic GB-aware sequential and batch channel assignment schemes that aim at maximizing the spectrum efficiency. In line with recent IEEE 802.11 and LTE standards, our schemes support bonding and aggregation. We propose two assignment models for each of the sequential and batch schemes: a static single-stage and an adaptive two-stage. In the static model, channel assignment is performed once such that the rate demands are probabilistically met. The adaptive model is a two-stage model, where the initial assignment may be corrected once uncertainties are partially revealed. We refer to our formulations of the sequential and batch static assignments as *chance-constrained stochastic subset-sum problem (CSSP)* and *chance-constrained stochastic multiple subset-sum problem (CMSSP)*, respectively. Moreover, we develop stochastic formulations for the sequential and batch adaptive assignments, which we refer to as *two-stage CSSP with recourse (CSSPR)* and *two-stage CMSSP with recourse (CMSSPR)*, respectively. Finally, we present computationally efficient simplified versions of CSSP and CSSPR with near-optimal performance.

Index Terms—Channel assignment, dynamic spectrum access, guard bands, multiple subset-sum problem, spectrum efficiency, stochastic optimization.

I. INTRODUCTION

MOTIVATED by the need for more efficient utilization of the licensed spectrum, and supported by recent regulatory policies (e.g., [2]), significant research has been conducted on dynamic spectrum access (DSA) networks. In a DSA system, secondary users (SUs) access the available spectrum in a dynamic and opportunistic fashion, without interfering with co-located incumbent users, i.e., primary users (PUs).

Manuscript received March 26, 2014; revised September 24, 2014 and January 15, 2015; accepted February 7, 2015. This work was supported by NPRP grant # NPRP 4-1034-2-385 from the Qatar National Research Fund (a member of Qatar Foundation). The statements made herein are solely the responsibility of the authors. A preliminary version of this paper was presented at IEEE GLOBECOM, December 2013. The associate editor coordinating the review of this paper and approving it for publication was F. Gao.

M. J. Abdel-Rahman was with the University of Arizona, Tucson, AZ 85721 USA. He is now with the Department of Electrical and Computer Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061 USA (e-mail: mjabdelrahman@email.arizona.edu).

M. Krunz is with the Department of Electrical and Computer Engineering, University of Arizona, Tucson, AZ 85721 USA (e-mail: krunz@email.arizona.edu).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TWC.2015.2413964

Channel quality in DSA networks is uncertain. This is partially due to inherent multi-path fading and shadowing, and partially to the unpredictability of PU activity. Recently, the FCC advocated using a database (DB) to facilitate DSA. According to this approach, an SU acquires the set of available channels in its geographical area through a centralized DB. The DB is mostly concerned with SU-to-PU interference, but not PU-to-SU interference [3]. It declares a channel to be available at a given location if the PU signal transmitted over this channel cannot be successfully decoded by a PU receiver at that location. As indicated in [4], the decodability threshold for a digital TV signal is ~ 15 dB, which is ~ 32 times stronger than the noise level. Therefore, even if a channel is declared available by the DB, this does not mean that it is completely clean, and it can still have a substantial “pollution” due to PU transmissions. In other words, PU dynamics causes the channel quality of a DSA network to be uncertain.

In addition to channel uncertainty, we consider two aspects: (i) accounting for adjacent-channel interference (ACI), and (ii) supporting channel bonding and aggregation. ACI is a form of power leakage that is attributed to imperfect filters and amplifiers in the radio device. The harmful impact of ACI on the throughput of IEEE 802.11a and IEEE 802.11n networks was demonstrated in [5] and [6], respectively. Most channel assignment algorithms in the literature do not account for ACI. To mitigate ACI, *guard-bands* (GBs) are needed between adjacent channels that belong to different links. Introducing GBs constrains spectrum efficiency. We aim at maximizing spectrum efficiency while mitigating ACI.

Channel bonding and aggregation enables the support of applications with high rate demands. Channel bonding refers to the bundling of adjacent channels, which can then be treated as a single frequency “block” whose data rate is approximately the sum of the data rates of the individual channels. Channel bonding has been adopted in recent IEEE 802.11n and IEEE 802.11ac standards [7]–[11]. It can be extended to non-adjacent channels, a concept referred to as *channel aggregation*. For example, LTE-Advanced employs channel aggregation techniques, allowing 4G mobile operators to aggregate spectrum from non-adjacent bands to support links with high demands [12]. Several resource allocation schemes with channel bonding (e.g., [13]–[15]) and aggregation (e.g., [16]–[18]) were proposed in the literature. However, none of these schemes account for ACI through GBs.

Recently, Uyanik *et al.* [19] proposed a GB-aware (GBA) channel assignment scheme that achieves optimal spectrum efficiency and supports channel bonding and aggregation.

However, the transmission rates of various channels were assumed to be deterministically known. Our objective in this paper is to design GBA channel assignment schemes for DSA networks, where channel quality is uncertain. It has been shown in [19], [20] that to attain optimal spectrum efficiency, channels need to be assigned on a per-block basis (as will be explained in Section II). Given this, the channel assignment problem in [19] can be restated as follows: Given the set of idle frequency blocks, obtain a combination of idle frequency blocks that either satisfies the link demand or achieves the nearest rate to this demand. This is exactly the subset-sum problem (SSP) [21].¹

A few stochastic SSP formulations were proposed in the literature to tackle the SSP under the uncertainty of the item weights and/or the knapsack capacity. Existing stochastic SSP formulations are not suitable for our stochastic GBA channel assignment problems. Therefore, using stochastic optimization theory, in this paper we develop new stochastic SSP formulations. To the best of our knowledge, none of the existing channel assignment schemes in the literature supports channel bonding, aggregation, and GBs, while simultaneously accounting for the channel quality uncertainty.

Our Contributions: To account for the uncertainty in channel quality, we develop novel stochastic GBA channel assignment schemes that maximize the spectrum efficiency and support channel bonding and aggregation. We propose both sequential (i.e., per-link) as well as batch (multi-link) channel assignment schemes. For each, we formulate two stochastic channel assignment models: *a static single-stage* and *an adaptive two-stage*. In the static model, channel assignment is performed once such that the given rate demands are met with a probability greater than a certain threshold. This model is appropriate for distributed systems that do not have a centralized spectrum manager. The adaptive model is a two-stage assignment model, where the initial assignment may be corrected once the uncertainties are *partially* revealed. This model is suitable for systems with a centralized spectrum manager.

We develop two stochastic SSP formulations for the sequential and batch static GBA channel assignment problems. We refer to these formulations as *chance-constrained stochastic SSP (CSSP)* and *chance-constrained stochastic multiple SSP (CMSSP)*, respectively. The “chance constraint” is introduced to restrict the probability of under-satisfying the link demand. We solve CSSP and CMSSP by deriving their deterministic equivalent formulations, which turn out to be binary integer linear programs (BILPs). Furthermore, we propose two formulations for the sequential and batch adaptive GBA channel assignment problems. We refer to these formulations as *two-stage CSSP with recourse (CSSPR)* and *two-stage CMSSP with recourse (CMSSPR)*, respectively. The second stage in CSSPR and CMSSPR is introduced to prevent link over-satisfaction, and also to improve the rate of the under-satisfied links in the first stage. Similar to CSSP and CMSSP, we solve CSSPR and CMSSPR by deriving their deterministic equivalent formulations, which also turn out to be BILPs. Finally, we develop computationally efficient simplified versions of CSSP and CSSPR,

¹Note that if aggregation is not allowed, the assignment problem becomes much easier, and it can be efficiently solved using the Hungarian method.

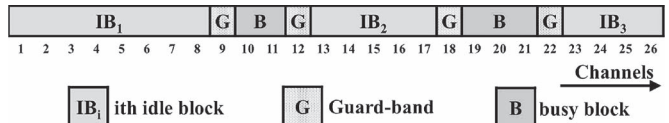


Fig. 1. Example of a spectrum status ($M = 26$ and $N = 3$). Channels 10–11 and 19–21 are busy. Two GBs are allocated around each block of busy channels, one on each side. After allocating the GBs, three blocks of idle channels are left: channels 1–8, 13–17, and 23–26.

and compare their performance with the exact deterministic equivalent formulations.

Paper Organization: The rest of the paper is organized as follows. In Section II, we provide the system model followed by the problem statement. The sequential static and adaptive channel assignment problems are formulated in Section III-A, and reformulated as BILPs in Section III-B. In Section IV-A, we formulate the batch static and adaptive channel assignment problems. The batch assignment problems are reformulated as BILPs in Section IV-B. In Section V, we present our numerical results. We conclude the paper in Section VI and provide directions for future research.

II. PROBLEM STATEMENT

Consider a DSA network² that consists of L links and operates over M licensed channels. An idle (i.e., PU free) channel can be used as a GB if it is adjacent to a busy channel, or as a data channel otherwise. A channel is considered busy if it is occupied by a PU or previously admitted SU. Assume that the current set of idle channels are grouped into N frequency blocks (after reserving the needed GBs). Each block consists of contiguous idle channels. Let $\mathcal{N} \stackrel{\text{def}}{=} \{1, 2, \dots, N\}$ and let $\tilde{R}_i, i \in \mathcal{N}$, be a random variable representing the data rate supported by the i th idle frequency block,³ denoted by IB_i . Let $\mathcal{L} \stackrel{\text{def}}{=} \{1, 2, \dots, L\}$ and let $d_j, j \in \mathcal{L}$, be the rate demand of link j in Mbps. Given the current spectrum status, i.e., the state of each of the M licensed channels, our objective is to satisfy the rate demands of the L links while maximizing the spectrum efficiency, defined as the fraction of the available spectrum that can be used for data communications. Fig. 1 shows an example of a spectrum status. The assignment that maximizes spectrum efficiency is the one that minimizes the number of introduced GBs.

Theorem 1: To maximize spectrum efficiency, channels need to be assigned on a per-block basis instead of per-channel basis [19].

Proof: See [19]. ■

In this paper, we study the following two problems:

Problem 1 (Sequential Assignment): Given an arbitrary link with a rate demand d Mbps, the current status of the M channels, and the rate distributions of the idle channels, find the

²The treatment in this paper applies to any wireless network. Considering a DSA network only affects the calculation of the rate distribution (which depends on the PU activity pattern). Because PU-to-SU interference is usually overlooked, channel-quality uncertainty is particularly important in DSA networks.

³The rate distribution of an idle frequency block is obtained from the rate distribution of the idle channels that constitute this block.

optimal GBA channel assignment for this link that maximizes the spectrum efficiency while satisfying the demand d .

Problem 2 (Batch Assignment): Given the set of links \mathcal{L} and their associated rate demands, the current status of the M channels, and the rate distributions of the idle channels, find the optimal GBA channel assignment that maximizes the spectrum efficiency while satisfying the link demands.

As mentioned earlier, our proposed assignment schemes support channel bonding and aggregation. In our formulations, we do not impose a limit on the total bonded/aggregated bandwidth. Although current wireless systems impose such a limit, there are several ongoing efforts to develop systems that support wide-band aggregation. The trend in the current IEEE 802.11/LTE standards is to increase the supported data rate by enabling bonding/aggregation of more channels.

We study Problems 1 and 2 considering two different system models. In the first model, channel block-assignment is performed once and cannot be corrected after the actual transmission rates become known to the assignment algorithm. This corresponds to a network setup where channel assignment is performed for each link independent of other links. The second model allows for adjusting the block assignment after the randomness is partially revealed, when nodes start communicating over the assigned blocks. According to the second model, the additional assigned blocks (if any) can be taken from over-satisfied links and reassigned to under-satisfied links. This dynamic assignment approach increases the number of admitted links in a resource-constrained network, operating under channel uncertainty.

III. SEQUENTIAL ASSIGNMENT

A. Problem Formulation

In this section, we formulate the sequential (per-link) channel block-assignment problem. Because the rates of idle blocks are random, assigning blocks based on their expected rates only may result in link under-satisfaction (when the actual rates are lower than their expected values) or link over-satisfaction (when the actual rates are higher than their expected values). To handle the uncertainty associated with these rates, we propose two different models based on stochastic programming techniques. The first model introduces a ‘‘chance constraint’’ to limit the likelihood of under-satisfaction, whereas the second model goes a step further, allowing for some corrective action in case of over-satisfaction.

1) *Static Single-Stage Assignment:* Considering a single link, in this section we formulate the channel block-assignment problem assuming the static (non-corrective) model. In this model, the assignment is performed once, and correcting the assignment after observing the actual block rates is not allowed. As mentioned earlier, the optimal channel block-assignment problem that maximizes spectrum efficiency was formulated in [19] as an SSP, assuming deterministic channel rates. When the channel rates are random, the feasible region of the assignment problem becomes uncertain. Different stochastic optimization approaches have been proposed in the literature to deal with the uncertainty of the feasibility region of an opti-

mization problem [22]. In here, we adopt a ‘‘chance constraint approach.’’

We formulate the static stochastic assignment problem under rates uncertainty as a chance-constrained stochastic SSP (CSSP) with a discrete distribution. Let $x_i, i \in \mathcal{N}$, be a binary variable; $x_i = 1$ if the i th frequency block is to be assigned, and zero otherwise. Then, the CSSP formulation is given by:

Problem CSSP:

$$\text{minimize}_{\{x_i, i \in \mathcal{N}\}} \sum_{i=1}^N \mu_i x_i \quad (1)$$

$$\text{subject to} \quad \Pr \left\{ \sum_{i=1}^N \tilde{R}_i x_i \geq d \right\} \geq \beta \quad (2)$$

$$x_i \in \{0, 1\}, \forall i \in \mathcal{N} \quad (3)$$

where $\mu_i \stackrel{\text{def}}{=} \mathbb{E}[\tilde{R}_i]$ and $\beta \in (0, 1]$ is a given probability. The objective (1) is to minimize the total expected rate of the assigned blocks, and the chance constraint (2) enforces satisfying the link demand with probability $\geq \beta$. While the chance constraint probabilistically accounts for link under-satisfaction, it does not hedge against the problem of link over-satisfaction. In a resource-constrained network with multiple links operating in parallel, over-satisfying one link may result in under-satisfying other links in the network.

2) *Adaptive Two-Stage Assignment:* The adaptive (corrective) channel block-assignment problem is formulated in this section. The frequency blocks are initially assigned based on their expected rates while satisfying the chance constraint. After observing the actual rates supported by the assigned blocks, additional blocks (if any) are released and returned to the spectrum manager. The returned blocks can then be used by other links.

Let $\mathbf{x} = (x_1, x_2, \dots, x_N)$ and $\tilde{\mathbf{R}} = (\tilde{R}_1, \tilde{R}_2, \dots, \tilde{R}_N)$. Then, the adaptive stochastic assignment problem is formulated as a two-stage CSSP with recourse (CSSPR), as follows:

Problem CSSPR:

$$\text{minimize}_{\{x_i, i \in \mathcal{N}\}} \left\{ \sum_{i=1}^N \mu_i x_i + \mathbb{E} \left[h(\mathbf{x}, \tilde{\mathbf{R}}) \right] \right\} \quad (4)$$

$$\text{subject to} \quad \Pr \left\{ \sum_{i=1}^N \tilde{R}_i x_i \geq d \right\} \geq \beta \quad (5)$$

$$x_i \in \{0, 1\}, \forall i \in \mathcal{N} \quad (6)$$

where $h(\mathbf{x}, \tilde{\mathbf{R}})$ is the optimal value of the second-stage problem, which is given by:

$$\text{minimize}_{\{y_i, i \in \mathcal{N}\}} \left\{ - \sum_{i=1}^N \alpha_i \tilde{R}_i y_i \right\} \quad (7)$$

$$\text{subject to} \quad y_i \leq x_i, \forall i \in \mathcal{N} \quad (8)$$

$$\sum_{k=1}^N (x_k - y_k) \tilde{R}_k \geq d y_i, \forall i \in \mathcal{N} \quad (9)$$

$$y_i \in \{0, 1\}, \forall i \in \mathcal{N}. \quad (10)$$

$\alpha_i \in [0, 1), i \in \mathcal{N}$, is a discount factor. The second-stage decision variable $y_i, i \in \mathcal{N}$, equals one if block i is removed (i.e., unassigned after it was previously assigned) and zero otherwise. The objective of the second-stage problem in (7) is to maximize the total rate of the released extra blocks. We assume that the rate of any block $IB_i, i \in \mathcal{N}$, at the second-stage (i.e., when the block is released after it was previously assigned) is strictly smaller than its first-stage rate, i.e., $\alpha_i < 1$. This way, the later the block is used by a link the smaller the rate this block can support. Setting α_i to be strictly less than one for all $i \in \mathcal{N}$ avoids having an undesirable aggressive assignment (when $\alpha_i = 1, \forall i \in \mathcal{N}$), in which a single link reserves all available blocks and then releases the additional ones. This approach is undesirable because all resources will be allocated to one link first, then all the surplus blocks will be allocated to another link, and so on. Constraint (8) enforces that only blocks that have been assigned in the first stage may be removed, and constraint (9) ensures that a block can be removed only when the first-stage assignment has led to an over-satisfaction, and that the link demand remains satisfied after the removal. As in CSSP, a chance constraint is introduced in the first stage to restrict the probability of under-satisfaction.

We note that CSSPR has a relatively complete recourse, i.e., for every feasible first-stage decision x_i that satisfies (5) and (6), there exists a feasible solution to the second-stage problem under each scenario $\omega \in \Omega$, where Ω is the set of ‘‘scenarios,’’ various realizations of the rates of various blocks. For example, $y_i = 0, \forall i \in \mathcal{N}$, is always a feasible solution to the second-stage problem.

B. Problem Reformulation and Solution Approach

Our approach for solving the CSSP and CSSPR formulations is to derive their deterministic equivalent programs (DEPs). The DEP is an equivalent reformulation of the original stochastic program, but contains only deterministic variables [22]. In this section, we first present the DEPs of CSSP and CSSPR. Because of their high complexity, in this section we also propose simplified versions of the CSSP and CSSPR formulations, which we refer to as CSSP-modified and CSSPR-modified, respectively.

1) Static Single-Stage Assignment:

a) *DEP*: To obtain the DEP of CSSP, we need to reformulate the chance constraint (2), so that it does not include the probability term or the random variables $\tilde{R}_i, i \in \mathcal{N}$. Let $p^{(\omega)}$ be the probability of scenario $\omega \in \Omega$. $p^{(\omega)}$ is considered as an input to our assignment problems. To reformulate the chance constraint, we will introduce a binary variable $u^{(\omega)}$ for each scenario $\omega \in \Omega$; $u^{(\omega)} = 0$ if the block assignment needs to satisfy the demand d under scenario ω , and one otherwise. Then, the chance constraint (2) is equivalent to the following two constraints:

$$\sum_{i=1}^N R_i^{(\omega)} x_i \geq d (1 - u^{(\omega)}), \quad \forall \omega \in \Omega \quad (11)$$

$$\sum_{\omega \in \Omega} p^{(\omega)} u^{(\omega)} \leq 1 - \beta. \quad (12)$$

TABLE I
COMPLEXITY OF VARIOUS SSP SOLUTIONS

Algorithm	Complexity
Dynamic programming (DP)-based exact	$\mathcal{O}(Nd)$
Pseudo-polynomial ϵ -approximate	$\mathcal{O}(\frac{1}{\epsilon} N^2 \ln d)$
Polynomial-time greedy	$\mathcal{O}(N \log N)$

For each scenario $\omega \in \Omega$, if $u^{(\omega)} = 0$ then (11) reduces to $\sum_{i=1}^N R_i^{(\omega)} x_i \geq d$ (i.e., the link demand d is satisfied under scenario ω). On the other hand, if $u^{(\omega)} = 1$, then (11) reduces to $\sum_{i=1}^N R_i^{(\omega)} x_i \geq 0$ which is a redundant constraint since the data rates of all blocks are always non-negative (i.e., $R_i^{(\omega)} \geq 0, \forall i \in \mathcal{N}, \forall \omega \in \Omega$). Constraint (12) basically says that the sum of the probabilities of the scenarios under which the link demand may not be satisfied (i.e., $\omega : u^{(\omega)} = 1$) is less than or equal to $1 - \beta$, which gives the same meaning as the chance constraint (2). We refer to the DEP of CSSP (after replacing (2) by (11) and (12)) by CSSP-DEP. We remark that CSSP-DEP is a binary integer linear program (BILP). The complexity of solving CSSP-DEP increases with the number of scenarios. Constraint (11) is repeated for each scenario $\omega \in \Omega$. $|\Omega|$ increases exponentially with the number of idle frequency blocks (i.e., N). For example, if we assume that each idle frequency block can support one of eight rates (as in IEEE 802.11a) according to a certain probability distribution, then $|\Omega| = 8^N$. To reduce the complexity of CSSP, in the following we propose a simplified version of CSSP.

b) *CSSP-modified*: Based on Markov’s inequality [23], the left-hand-side of the chance constraint (2) can be bounded from above as follows:

$$\Pr \left\{ \sum_{i=1}^N \tilde{R}_i x_i \geq d \right\} \leq \frac{\mathbb{E} \left[\sum_{i=1}^N \tilde{R}_i x_i \right]}{d} = \frac{\sum_{i=1}^N \mu_i x_i}{d}. \quad (13)$$

Hence, if constraint (2) is satisfied, the following inequality is also satisfied:

$$\sum_{i=1}^N \mu_i x_i \geq d\beta. \quad (14)$$

However, if (14) is satisfied, this does not necessarily imply that (2) is satisfied. Therefore, we make (14) more stringent by multiplying the right-hand-side of (14) with a constant $\kappa > 1$, so that constraint (14) becomes:

$$\sum_{i=1}^N \mu_i x_i \geq \kappa d\beta. \quad (15)$$

The value of κ is determined empirically such that (15) best approximates the chance constraint (2). If the chance constraint (2) is replaced with (15), CSSP becomes a standard *deterministic* SSP. In this SSP, the value of the i th frequency block is μ_i and the demand is $\kappa d\beta$. Although SSP is still an NP-hard problem, there exist a pseudo-polynomial ϵ -approximate algorithm and a polynomial-time greedy heuristic for solving it. Table I lists the complexity of various SSP solutions. Our procedure for solving the modified version of CSSP is summarized in Algorithm 1.

Algorithm 1 CSSP-modified

- 1: **Input:** $N, d, \beta, \kappa, \mu_i, i \in \mathcal{N}, p^{(\omega)}, \omega \in \Omega$
 - 2: **Output:** $x_i, i \in \mathcal{N}$, or declare the problem as infeasible
 - 3: Replace (2) in CSSP with (15), and solve the resulting (deterministic) SSP
 - 4: Evaluate (2) with respect to the obtained solution
 - 5: **If** (2) is satisfied **Then**
 - 6: Terminate, and report $x_i, i \in \mathcal{N}$
 - 7: **Else**
 - 8: Select more blocks from the unassigned ones, starting with the one that has the smallest expected rate until a solution to (2) is found. Then, terminate
 - 9: **End If**
 - 10: **If** assigning all blocks does not lead to a solution **Then**
 - 11: Declare the problem as infeasible
 - 12: **End If**
-

Constraint (2) is evaluated for a given block assignment as follows:

1. Sort the scenarios in a descending order according to their probabilities ($p^{(\omega)}$ for scenario $\omega \in \Omega$).
2. Starting from the most probable scenario, do the following:
 - Evaluate the link demand constraint $\sum_{i=1}^N R_i^{(\omega)} x_i \geq d$ under scenario $\omega \in \Omega$.
 - If scenario ω satisfies the link demand constraint, increment the probability of satisfying the link demand by $p^{(\omega)}$.
 - If the total probability of satisfying the link demand reaches or exceeds β , then the given block assignment satisfies (2). Otherwise, go to the next most probable scenario.
3. If all the scenarios have been considered and the total probability of satisfying the link demand is less than β , then the given block assignment does not satisfy (2).

It is to be mentioned that κ is a critical parameter of CSSP-modified. If κ is set to a very large value, then (15) might not be satisfied. On the other hand, if κ is set to a very small value, then the solution of the deterministic SSP might not be a feasible solution to CSSP. The value of κ that makes (15) best approximates (2) depends on the other system parameters, such as the number of data blocks, d , and β .

2) *Adaptive Two-Stage Assignment:*

a) *DEP:* To obtain the DEP of CSSPR, we first reformulate the first-stage chance constraint (5), as in CSSP. Then, the second-stage objective function (7) is substituted in (4). Then, the DEP of the CSSPR is given by:

Problem CSSPR-DEP:

$$\left\{ \begin{array}{l} \text{minimize} \\ x_i, y_i^{(\omega)}, u^{(\omega)} \\ i \in \mathcal{N}, \omega \in \Omega \end{array} \right\} \left\{ \sum_{i=1}^N \mu_i x_i - \sum_{\omega \in \Omega} p^{(\omega)} \left(\sum_{i=1}^N \alpha_i R_i^{(\omega)} y_i^{(\omega)} \right) \right\} \quad (16)$$

subject to (11)–(12)

$$y_i^{(\omega)} \leq x_i, \forall i \in \mathcal{N}, \forall \omega \in \Omega \quad (17)$$

$$\sum_{k=1}^N \left(x_k - y_k^{(\omega)} \right) R_k^{(\omega)} \geq d y_i^{(\omega)}, \forall i \in \mathcal{N}, \forall \omega \in \Omega \quad (18)$$

$$x_i, y_i^{(\omega)}, u^{(\omega)} \in \{0, 1\}, \forall i \in \mathcal{N}, \forall \omega \in \Omega. \quad (19)$$

The second term in the objective function (16) is the expected value of the objective function of the second-stage problem of CSSPR (i.e., (7)). Again, CSSPR-DEP is a BILP.

b) *CSSPR-modified:* The simplified version of CSSPR is summarized in Algorithm 2. In line 3 of Algorithm 2, we obtain a feasible first-stage solution to CSSPR from CSSP-modified (Algorithm 1). A feasible solution to CSSP is always feasible to CSSPR, because, as we mentioned earlier, CSSPR has a relatively complete recourse.

In line 9 of Algorithm 2, for a fixed first-stage solution $x_i, i \in \mathcal{N}$, and a specific scenario $\omega \in \Omega$, the second-stage problem reduces to a *deterministic* SSP, where only the blocks that have been assigned in the first stage are considered in this SSP. This can be seen as follows:

- Constraint (8) reduces to $y_i \leq 1, \forall i \in \mathcal{N}$, which is redundant, given constraint (10).
- Constraint (9) can be rearranged as:

$$\sum_{\substack{k=1 \\ k \neq i}}^N R_k^{(\omega)} y_k + \left(R_i^{(\omega)} + d \right) y_i \leq \sum_{k=1}^N R_k^{(\omega)} x_k. \quad (20)$$

Note that the right-hand-side of (20) is constant.

Algorithm 2 CSSPR-modified

- 1: **Input:** $N, d, \beta, \kappa, \mu_i, p^{(\omega)}, \omega \in \Omega, R_i^{(\omega)}, i \in \mathcal{N}, \omega \in \Omega$
 - 2: **Output:** $x_i, y_i^{(\omega)}, i \in \mathcal{N}, \omega \in \Omega$, or declare the problem as infeasible
 - 3: Use Algorithm 1 to obtain a feasible $x_i, i \in \mathcal{N}$
 - 4: **If** there is no feasible $x_i, i \in \mathcal{N}$ **Then**
 - 5: Declare the problem as infeasible
 - 6: **End If**
 - 7: **For** each $\omega \in \Omega$ **Do**
 - 8: Fix $x_i, i \in \mathcal{N}$, as obtained from Algorithm 1
 - 9: Solve the second-stage problem for scenario ω , to determine the blocks to be excluded from the initial assignment
 - 10: **End For**
-

IV. BATCH ASSIGNMENT

A. Problem Formulation

Assigning channels to multiple links in a wireless network can be done sequentially, i.e., one link at a time. In this case, the “exact”⁴ and the modified assignment schemes proposed in Section III-B can be applied sequentially (note that the spectrum status needs to be updated after each link assignment, before executing the next link assignment). The links can be considered in different orders. In Section V, we implement

⁴By exact schemes, we mean the solutions of the CSSP-DEP and CSSPR-DEP optimization problems.

two ordering approaches: (i) assign channels to links in an ascending order of link demands (denoted by SEQ_{asc}), and (ii) assign channels in a descending order of link demands (denoted by SEQ_{dsc}).

However, sequential assignment (in general) does not achieve the optimal network-wide spectrum efficiency. Another way of assigning channels to multiple links is the *batch assignment*, in which all links are considered simultaneously. The batch assignment approach can be implemented in a centralized or a distributed manner. To implement the batch assignment in a distributed network, one may apply the *access window (AW)* concept used in [24] and [25], where each link broadcasts its rate demand in a given slot. Each link waits for a certain amount of time to collect the demands of other links in the network before executing the joint assignment problem.

We propose two batch assignment schemes: A static single-stage and an adaptive two-stage assignment schemes.

1) *Static Single-Stage Assignment*: In here, we extend the single-link static assignment scheme proposed in Section III-A1 to multiple links. Specifically, instead of a single chance constraint, we add a chance constraint for each link. Moreover, instead of the single-link expected rate, the objective function becomes the network-wide expected rate. Let $x_{ij}, i \in \mathcal{N}$ and $j \in \mathcal{L}$, be a binary variable; $x_{ij} = 1$ if block i is assigned to link j , and zero otherwise. Then, the static batch assignment problem is formulated as a chance-constrained stochastic multiple SSP (CMSSP) as follows:

Problem CMSSP:

$$\underset{\{x_{ij}, i \in \mathcal{N}, j \in \mathcal{L}\}}{\text{minimize}} \sum_{j=1}^L \sum_{i=1}^N \mu_i x_{ij} \quad (21)$$

$$\text{subject to } \Pr \left\{ \sum_{i=1}^N \tilde{R}_i x_{ij} \geq d_j \right\} \geq \beta_j, \forall j \in \mathcal{L} \quad (22)$$

$$\sum_{j=1}^L x_{ij} \leq 1, \forall i \in \mathcal{N} \quad (23)$$

$$x_{ij} \in \{0, 1\}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L} \quad (24)$$

where constraint (23) ensures that an idle block can be allocated to one link only.

Remark 1: In the above CMSSP formulation, links are assumed to interfere with each other. Therefore, constraint (23) ensures that an idle block cannot be assigned to more than one link at the same time. Our CMSSP formulation can be extended to accommodate scenarios where links do not necessarily conflict with each other. To do this, for each pair of links k and $m \in \mathcal{L}$, we introduce a binary variable y_{km} . $y_{km} = 1$ if links k and m are conflicting, and $y_{km} = 0$ otherwise. Then, constraint (23) is replaced by the following constraint:

$$x_{ik} + x_{im} \leq 2 - y_{km}, \forall i \in \mathcal{N}, \forall k, m \in \mathcal{L}. \quad (25)$$

2) *Adaptive Two-Stage Assignment*: The adaptive batch assignment scheme is formulated as a two-stage CMSSP with recourse (CMSSPR), which consists of two stages. The first stage is similar to CMSSP. The second stage of CMSSPR is more complicated than the second stage of CSSPR. In the second stage of CSSPR, blocks can only be removed (if they

are excess). In the second-stage of CMSSPR, excess blocks can be removed from over-satisfied links, and also they can be added to under-satisfied links, which makes the formulation more involved.

In addition to maximizing the discounted sum-rate of the blocks that can be taken from over-satisfied links (as in CSSPR), the second stage of CMSSPR aims to: (i) maximize the discounted sum-rate of the blocks that can be added to the under-satisfied links, and (ii) minimize the sum (over all links) of the difference between the link demand and its assigned rate. Let $x_{ij}, i \in \mathcal{N}, j \in \mathcal{L}$, be defined as in CMSSP. Let $y_{ij}, i \in \mathcal{N}, j \in \mathcal{L}$, be a binary variable; $y_{ij} = 1$ if block i is removed from link j , and zero otherwise. Also, let $z_{ij}, i \in \mathcal{N}, j \in \mathcal{L}$, be a binary variable; $z_{ij} = 1$ if block i is added to link j , and zero otherwise. Then, the objective function of the second stage of CMSSPR can be expressed as follows:

$$\underset{\left\{ \begin{array}{l} y_{ij}, z_{ij} \\ i \in \mathcal{N}, j \in \mathcal{L} \end{array} \right\}}{\text{minimize}} \left\{ - \sum_{j=1}^L \sum_{i=1}^N \alpha_i \tilde{R}_i (y_{ij} + z_{ij}) + \sum_{j=1}^L \left| d_j - \sum_{i=1}^N (x_{ij} - y_{ij} + z_{ij}) \tilde{R}_i \right| \right\} \quad (26)$$

where α_i is defined as in CSSPR. The term $\sum_{i=1}^N (x_{ij} - y_{ij} + z_{ij}) \tilde{R}_i$ represents the total rate assigned to link j . We can remove the absolute value and simplify (26) by introducing an auxiliary variable for each link. Let $e_j, j \in \mathcal{L}$, be the auxiliary variables. Then (26) can be replaced by:

$$\underset{\left\{ \begin{array}{l} y_{ij}, z_{ij}, e_j \\ i \in \mathcal{N}, j \in \mathcal{L} \end{array} \right\}}{\text{minimize}} \left\{ - \sum_{j=1}^L \sum_{i=1}^N \alpha_i \tilde{R}_i (y_{ij} + z_{ij}) + \sum_{j=1}^L e_j \right\} \quad (27)$$

$$\text{subject to } e_j \geq d_j - \sum_{i=1}^N (x_{kj} - y_{kj} + z_{kj}) \tilde{R}_k, \forall j \in \mathcal{L} \quad (28)$$

$$e_j \geq \sum_{i=1}^N (x_{kj} - y_{kj} + z_{kj}) \tilde{R}_k - d_j, \forall j \in \mathcal{L}. \quad (29)$$

The second-stage problem of CMSSPR includes the following constraints:

1. A block can be released only if it has been already assigned.
2. A block can be taken only from over-satisfied links.
3. A block can be assigned to under-satisfied links only.
4. A block can be assigned to an under-satisfied link only if it can be released from an over-satisfied link.
5. A released block from an over-satisfied link cannot be reassigned to the same link.
6. A released block can be assigned only to one under-satisfied link.

The first two constraints are the same as in CSSPR, and can be enforced by adding:

$$y_{ij} \leq x_{ij}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L} \quad (30)$$

$$\sum_{k=1}^N (x_{kj} - y_{kj}) \tilde{R}_k \geq d_j y_{ij}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}. \quad (31)$$

Constraint 3 above can be ensured by adding:

$$\sum_{k=1}^N (x_{kj} - y_{kj} + z_{kj}) \tilde{R}_k \geq d_j \eta_{ij} + z_{ij} \tilde{R}_i, \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{L} \quad (32)$$

$$z_{ij} \leq 1 - \eta_{ij}, \eta_{ij} \in \{0, 1\}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L} \quad (33)$$

Note that if the link demand is already met, η_{ij} will be one which enforces z_{ij} to be zero. On the other hand, if the link demand is not satisfied, η_{ij} will be zero, and in this case z_{ij} can be zero or one. Constraint 4 can be met by imposing:

$$z_{ij} \leq \sum_{k=1}^L y_{ik}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}. \quad (34)$$

Finally, to ensure satisfying constraints 5 and 6, we add the following two constraints, respectively:

$$z_{ij} \leq 1 - y_{ij}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L} \quad (35)$$

$$\sum_{j=1}^L z_{ij} \leq 1, \forall i \in \mathcal{N}. \quad (36)$$

The resulting CMSSPR formulation of the adaptive batch assignment is given by:

Problem CMSSPR:

$$\begin{aligned} & \underset{\{x_{ij}, i \in \mathcal{N}, j \in \mathcal{L}\}}{\text{minimize}} \left\{ \sum_{j=1}^L \sum_{i=1}^N \mu_i x_{ij} + \mathbb{E} \left[h(\mathbf{x}, \tilde{\mathbf{R}}) \right] \right\} \\ & \text{subject to} \quad (22)\text{--}(24) \end{aligned} \quad (37)$$

where $h(\mathbf{x}, \tilde{\mathbf{R}})$ is the optimal value of the second-stage problem, which is given by:

$$\begin{aligned} & \underset{\left\{ \begin{array}{l} y_{ij}, z_{ij} \\ \eta_{ij}, e_j \\ i \in \mathcal{N}, j \in \mathcal{L} \end{array} \right\}}{\text{minimize}} \left\{ - \sum_{j=1}^L \sum_{i=1}^N \alpha_i \tilde{R}_i (y_{ij} + z_{ij}) + \sum_{j=1}^L e_j \right\} \\ & \text{subject to} \quad (28)\text{--}(36). \end{aligned} \quad (38)$$

Remark 2: In our CMSSPR formulation, all extra blocks are removed from over-satisfied links, even if they will not be used by other links during the current AW, i.e., we can have a situation where $y_{ij} = 1$ and $z_{ij'} = 0$, $\forall j' \neq j$. The reason is that new demands may arrive during the next AW, which will use the excess blocks from the previous AW.

B. Problem Reformulation and Solution Approach

Following the same techniques used in Section III-B, we derive in this section the DEPs of CMSSP and CMSSPR.

1) *Static Single-Stage Assignment:* The DEP of CMSSP is given by:

Problem CMSSP-DEP:

$$\begin{aligned} & \underset{\left\{ \begin{array}{l} x_{ij}, u_j^{(\omega)} \\ i \in \mathcal{N}, j \in \mathcal{L}, \omega \in \Omega \end{array} \right\}}{\text{minimize}} \sum_{j=1}^L \sum_{i=1}^N \mu_i x_{ij} \\ & \text{subject to} \quad \sum_{i=1}^N R_i^{(\omega)} x_{ij} \geq d_j \left(1 - u_j^{(\omega)} \right), \forall j \in \mathcal{L}, \forall \omega \in \Omega \end{aligned} \quad (39)$$

(40)

TABLE II
RATE DISTRIBUTION OF FIVE FREQUENCY BLOCKS USED IN THE SINGLE-LINK ASSIGNMENT SCHEMES ($r_1 = 0$ Mbps, $r_2 = 1$ Mbps, $r_3 = 2$ Mbps, $r_4 = 4$ Mbps, AND $r_5 = 6$ Mbps)

Idle block	Pr[r_1]	Pr[r_2]	Pr[r_3]	Pr[r_4]	Pr[r_5]
IB ₁	0.1	0.8	0.1	0.0	0.0
IB ₂	0.05	0.1	0.7	0.1	0.05
IB ₃	0.0	0.05	0.4	0.5	0.05
IB ₄	0.0	0.05	0.1	0.8	0.05
IB ₅	0.0	0.0	0.1	0.4	0.5

$$\sum_{\omega \in \Omega} p^{(\omega)} u_j^{(\omega)} \leq 1 - \beta_j, \forall j \in \mathcal{L} \quad (41)$$

$$\sum_{j=1}^L x_{ij} \leq 1, \forall i \in \mathcal{N} \quad (42)$$

$$x_{ij}, u_j^{(\omega)} \in \{0, 1\}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}, \forall \omega \in \Omega. \quad (43)$$

2) *Adaptive Two-Stage Assignment:* The DEP of CMSSPR is given by (44)–(52), shown at the bottom of the next page.

To reduce the complexity of CMSSP and CMSSPR, links may be considered sequentially according to a certain order. For each link assignment, CSSP-modified and CSSPR-modified can be used for the static and adaptive models, respectively. We refer to the resulting schemes as CMSSP-modified and CMSSPR-modified, respectively.

V. NUMERICAL EVALUATION

In this section, we numerically evaluate the performance of our proposed assignment schemes. All schemes are implemented in CPLEX.

A. Single-Link Assignment

In this part, we evaluate our proposed single-link assignment schemes (CSSP, CSSP-modified, CSSPR, and CSSPR-modified). We compare CSSP with CSSPR. Furthermore, we study the performance of the modified CSSP and CSSPR schemes relative to the optimal CSSP and CSSPR schemes.

We set $N = 5$. Each frequency block can support one of the following rates: $r_1 = 0$ Mbps,⁵ $r_2 = 1$ Mbps, $r_3 = 2$ Mbps, $r_4 = 4$ Mbps, and $r_5 = 6$ Mbps, with the probability distribution shown in Table II. α_i is set to 0.8 for all $i \in \mathcal{N}$. We solve CSSP and CSSPR for different combinations of β and d values. Our proposed CSSP-modified and CSSPR-modified algorithms produce upper bounds for both models. We set $\kappa = 1.5$ in (15), as it is found to be efficient for most instances.

Fig. 2 depicts the optimal expected link throughput of CSSP and CSSPR versus β for $d = 6$ and 10 Mbps. The figure shows that for $\beta \geq 0.7$, the optimal expected link throughput of both CSSP and CSSPR generally exceeds the demand. Moreover, Fig. 2 shows that the expected link throughput of CSSP increases with β . Increasing β makes the chance constraint in (2) more stringent, resulting in over-satisfying the link demand. In

⁵This corresponds to the case when the block is occupied by PUs.

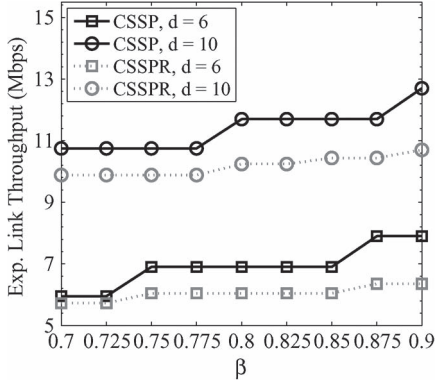


Fig. 2. Expected link throughput vs. β for CSSP and CSSPR (single-link assignment).

contrast, the expected throughput of CSSPR does not increase significantly with β . This is due to the recourse action in the second stage of CSSPR, in which we attempt to remove additional blocks. This results in a smaller net link throughput, leaving more channels to the links that will be subsequently assigned. This leads to increasing the admission rate of the sequential assignment schemes.

Fig. 3 shows the expected link throughput of CSSP and CSSP-modified vs. d for $\beta = 0.7, 0.8$, and 0.9 . Similarly, Fig. 4 shows the expected link throughput of CSSPR and CSSPR-modified. As can be seen, for large values of β and d both CSSP and CSSPR are infeasible. Also, these figures show that CSSP-modified and CSSPR-modified achieve a relatively close-to-optimal performance, and at some instances the

solutions of CSSP and CSSP-modified coincide (similarly for CSSPR and CSSPR-modified).

B. Multi-Link Assignment

In this part, we consider multiple links. We study both the sequential as well as the batch assignment schemes. We set $N = 8$. Each frequency block can support one of the following rates: $r_1 = 0$ Mbps, $r_2 = 1$ Mbps, and $r_3 = 4$ Mbps, with the probability distribution shown in Table V. $\alpha_i = 0.8, \forall i \in \mathcal{N}$.

1) *Sequential Assignment*: We implement two sequential assignment schemes, SEQ_{asc} and SEQ_{dsc} . In SEQ_{asc} , links are ordered in an ascending order of their rate demands, then considered one at a time. In contrast, links are ordered in a descending order of their demands in the SEQ_{dsc} scheme. To compare SEQ_{asc} and SEQ_{dsc} , we use the CSSP scheme in assigning each link. We set $L = 4$ and run SEQ_{asc} and SEQ_{dsc} for different values of β ($\beta_j = \beta, \forall j \in \mathcal{L}$). The rate demands of the four links are: $d_1 = 6$ Mbps, $d_2 = 4$ Mbps, $d_3 = 2.5$ Mbps, and $d_4 = 1.5$ Mbps. We evaluate the percentage of admitted links (i.e., the number of satisfied links divided by L) for different values of β . Fig. 5 shows the percentage of admitted links vs. β . As shown in the figure, the order of considering links matters, and the best way of ordering links depends on the value of β . When β equals 0.7 or 0.75 , if links are considered in a descending order of their demands then all of them will be admitted. However, if they are considered in an ascending order of their demands, only three links out of four will be admitted. On the other hand, when β equals 0.85 or 0.9 , only 50% of the links are admitted in SEQ_{dsc} compared to 75% of the links in SEQ_{asc} .

Problem CMSSPR-DEP:

$$\begin{aligned} & \underset{\{x_{ij}, y_{ij}^{(\omega)}, z_{ij}^{(\omega)}, u_j^{(\omega)}, e_j^{(\omega)}, \eta_{ij}^{(\omega)}\}}{\text{minimize}} \left\{ \sum_{j=1}^L \sum_{i=1}^N \mu_i x_{ij} - \sum_{\omega \in \Omega} p^{(\omega)} \left(\sum_{j=1}^L \sum_{i=1}^N \alpha_i R_i^{(\omega)} (y_{ij}^{(\omega)} + z_{ij}^{(\omega)}) - \sum_{j=1}^L e_j^{(\omega)} \right) \right\} \\ & \text{subject to (40)–(42)} \end{aligned} \quad (44)$$

$$e_j^{(\omega)} \geq d_j - \sum_{k=1}^N (x_{kj} - y_{kj}^{(\omega)} + z_{kj}^{(\omega)}) R_k^{(\omega)}, \forall j \in \mathcal{L}, \forall \omega \in \Omega \quad (45)$$

$$e_j^{(\omega)} \geq \sum_{k=1}^N (x_{kj} - y_{kj}^{(\omega)} + z_{kj}^{(\omega)}) R_k^{(\omega)} - d_j, \forall j \in \mathcal{L}, \forall \omega \in \Omega \quad (46)$$

$$y_{ij}^{(\omega)} \leq x_{ij}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}, \forall \omega \in \Omega \quad (47)$$

$$\sum_{k=1}^N (x_{kj} - y_{kj}^{(\omega)}) R_k^{(\omega)} \geq d_j y_{ij}^{(\omega)}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}, \forall \omega \in \Omega \quad (48)$$

$$\sum_{k=1}^N (x_{kj} - y_{kj}^{(\omega)} + z_{kj}^{(\omega)}) R_k^{(\omega)} \geq d_j \eta_{ij}^{(\omega)} + z_{ij}^{(\omega)} R_i^{(\omega)}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}, \forall \omega \in \Omega \quad (49)$$

$$z_{ij}^{(\omega)} \leq 1 - \eta_{ij}^{(\omega)}, z_{ij}^{(\omega)} \leq \sum_{k=1}^L y_{ik}^{(\omega)}, z_{ij}^{(\omega)} \leq 1 - y_{ij}^{(\omega)}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}, \forall \omega \in \Omega \quad (50)$$

$$\sum_{j=1}^L z_{ij}^{(\omega)} \leq 1, \forall i \in \mathcal{N}, \forall \omega \in \Omega \quad (51)$$

$$x_{ij}, u_j^{(\omega)}, y_{ij}^{(\omega)}, z_{ij}^{(\omega)}, \eta_{ij}^{(\omega)} \in \{0, 1\}, \forall i \in \mathcal{N}, \forall j \in \mathcal{L}, \forall \omega \in \Omega. \quad (52)$$

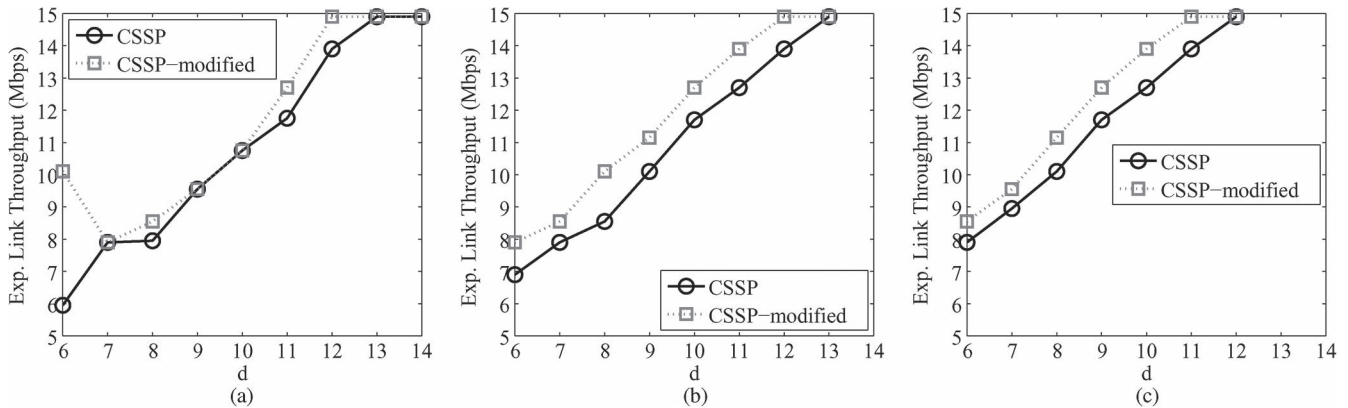


Fig. 3. Expected link throughput vs. d for CSSP (single-link assignment). (a) $\beta = 0.7$. (b) $\beta = 0.8$. (c) $\beta = 0.9$.

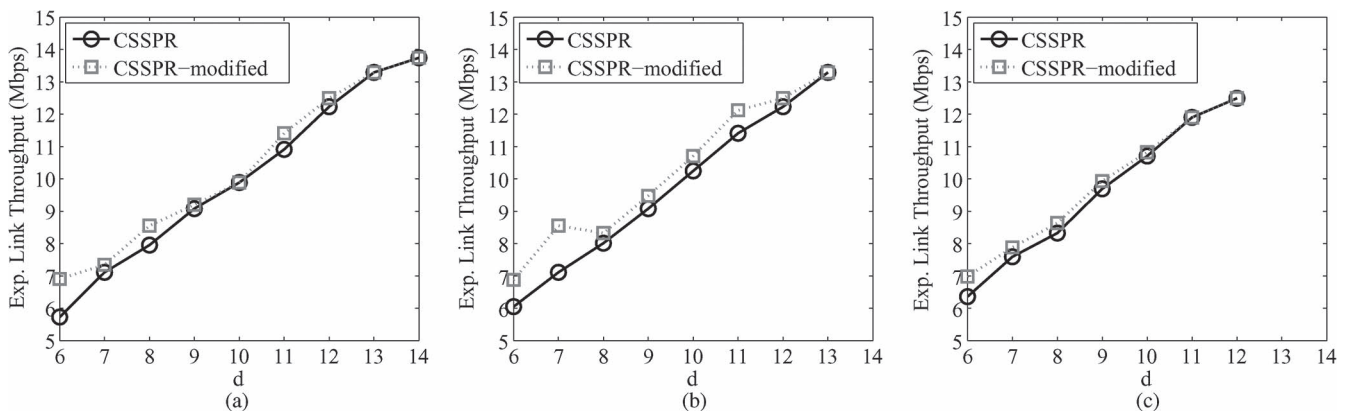


Fig. 4. Expected link throughput vs. d for CSSPR (single-link assignment). (a) $\beta = 0.7$. (b) $\beta = 0.8$. (c) $\beta = 0.9$.

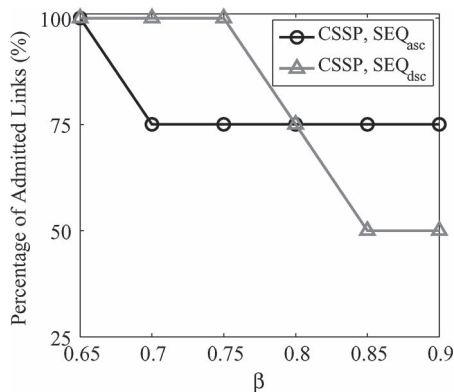


Fig. 5. Percentage of admitted links for sequential (CSSP-based) assignment schemes.

Because in the batch assignment all links are considered simultaneously, it achieves the best performance (i.e., in terms of the percentage of admitted links) of all sequential assignment schemes. In here, we implemented two sequential assignment schemes (SEQ_{asc} and SEQ_{dsc}) to illustrate the fact that the order of considering links is important. Moreover, the best order of considering links depends on β and the link demands. The batch assignment achieves an admission rate of 100% (if the available resources are sufficient to probabilistically satisfy the link demands), which may not be achieved by any sequential scheme for all values of β . This is because a sequential scheme

TABLE III
EXPECTED THROUGHPUT (IN Mbps) OF THE SINGLE-STAGE
MULTI-LINK ASSIGNMENT SCHEMES

$(\beta_1, \beta_2, \beta_3)$	(d_1, d_2, d_3)	Exp. Throughput (CMSSP)	Exp. Throughput (CMSSP-modified)
(0.7, 0.7, 0.7)	(7, 13, 14)	37.7	37.7
(0.8, 0.8, 0.8)	(7, 13, 13)	36.7	37.7
(0.9, 0.9, 0.9)	(8, 11, 12)	38.9	40.95

that is optimal for a given value of β may not be optimal for other values of β .

2) *Batch Assignment*: To show the advantage of the adaptive model (CMSSPR) compared to the static model (CMSSP), we set $L = 4$ and run CMSSP and CMSSPR for different values of β . The rate demands of the four links are: $d_1 = 2$ Mbps, $d_2 = 1$ Mbps, $d_3 = 1.5$ Mbps, and $d_4 = 1$ Mbps. We evaluate the probability of demand unsatisfaction (i.e., the sum of the probabilities of the scenarios under which the rate demand is not satisfied) of both CMSSP and CMSSPR for different values of β . Fig. 6 shows the probability of demand unsatisfaction for the first and third links as a function of β . The added second-stage in CMSSPR reduces the probability of demand unsatisfaction to almost zero. It redistributes the blocks between the links, trying to satisfy all demands. Note that constraint (22)

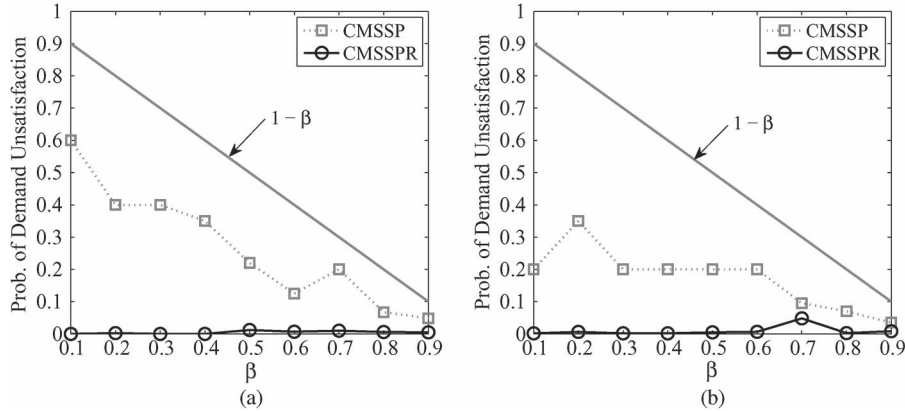


Fig. 6. Probability of demand unsatisfaction vs. β for links 1 and 3. (a) Link # 1 (demand = 2 Mbps). (b) Link # 3 (demand = 1.5 Mbps).

TABLE IV
EXPECTED THROUGHPUT (IN Mbps) OF THE TWO-STAGE
MULTI-LINK ASSIGNMENT SCHEMES

$(\beta_1, \beta_2, \beta_3)$	(d_1, d_2, d_3)	Exp. Throughput (CMSSPR)	Exp. Throughput (CMSSPR-modified)
(0.7, 0.7, 0.7)	(7, 13, 14)	34.1392	34.374
(0.8, 0.8, 0.8)	(7, 13, 13)	33.6872	35.13
(0.9, 0.9, 0.9)	(8, 11, 12)	32.71212	33.01922

TABLE V
RATE DISTRIBUTION OF EIGHT FREQUENCY BLOCKS USED
IN THE MULTI-LINK ASSIGNMENT SCHEMES

Idle block	$\Pr[r_1 = 0 \text{ Mbps}]$	$\Pr[r_2 = 1 \text{ Mbps}]$	$\Pr[r_3 = 4 \text{ Mbps}]$
IB ₁	0.1	0.3	0.6
IB ₂	0.2	0.4	0.4
IB ₃	0.0	0.7	0.3
IB ₄	0.05	0.15	0.8
IB ₅	0.25	0.35	0.4
IB ₆	0.05	0.75	0.2
IB ₇	0.0	0.5	0.5
IB ₈	0.15	0.2	0.65

enforces the probability of demand unsatisfaction of CMSSP to be smaller than $1 - \beta$. As shown in the figure, the probability of demand unsatisfaction reduces with β . Furthermore, this probability is higher for link 1 than link 3 because d_1 is greater than d_3 .

Finally, in Table III we compare the performance of CMSSP with CMSSP-modified. Similarly, in Table IV we compare the performance of CMSSPR with CMSSPR-modified. In Tables III and IV, we consider three links and 15 frequency blocks. The frequency blocks are classified into five categories with each category containing three blocks that have the same rate distribution (shown in Table II). In CMSSP-modified, the CSSP-modified scheme is applied to the three links sequentially in a descending order of their rate demands. As shown in Table III, the performance gap between CMSSP and CMSSP-modified varies with the values of β_j and d_j , $j \in \mathcal{L}$. In some cases, CMSSP and CMSSP-modified have the same performance. In CMSSPR-modified, the CSSPR-modified scheme is applied to the links sequentially in a descending order of their rate demands. Again, the performance gap between CMSSPR and CMSSPR-modified varies with the values of β_j and d_j , $j \in \mathcal{L}$.

VI. CONCLUSION AND FUTURE RESEARCH

In this paper, we studied the problem of performing GBA channel assignment with bonding and aggregation under channel-quality uncertainty. We considered both sequential as well as batch assignment schemes. For each scheme, we developed two assignment models: a static single-stage and an adaptive two-stage. All assignment schemes were formulated using

stochastic optimization techniques. Optimal solutions for the formulated assignment problems were obtained. Furthermore, we designed computationally efficient simplified stochastic assignment algorithms. We conducted numerical experiments to: (i) examine the efficiency of our simplified assignment schemes under different values of the model parameters, (ii) compare the performance of the static assignment with the adaptive assignment, and (iii) compare the sequential assignment schemes with the batch schemes. Our results indicate the following. First, the performance of the simplified assignment schemes is very close to the optimal, especially in the adaptive assignment. Second, the adaptive assignment is more powerful than the static assignment in uncertain environments. In particular, the probability of demand unsatisfaction of the adaptive assignment scheme is close to zero, whereas this probability can reach to 0.6 in the static scheme (depending on the system parameters). Third, the batch assignment achieves the best admission rate of all sequential assignment schemes. In our experiments (for four links), the admission rate of the batch scheme can reach up to 25% higher than that of any of the two sequential schemes, SEQ_{asc} and SEQ_{dsc}.

As future research, we aim to investigate the problem of designing efficient ϵ -approximate algorithms for solving our proposed stochastic assignment problems. As a first step, we plan to develop an ϵ -approximate algorithm for solving the CSSP formulation. To achieve this, we propose to start with the ϵ -approximate algorithm of the deterministic SSP [26]. Consider a particular combination of idle frequency blocks. Instead of computing the total rate that these blocks can support, as in the ϵ -approximate algorithm of the deterministic SSP, we need

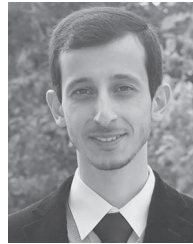
to compute the maximum rate that these blocks can support with probability greater than β . Such computation requires considering all rate scenarios that can happen and their probability of occurrence (the number of scenarios is exponential in the number of idle blocks). To address this computation complexity, we propose to design a ‘Trim’ function similar to the one used in the ϵ -approximate algorithm of the deterministic SSP [26] such that a large portion of the scenarios can be neglected. The trimming criterion will depend on the difference between the rates supported under various scenarios as well as the probabilities of these scenarios. Such line of investigation will be further explored in our future work.

ACKNOWLEDGMENT

The authors would like to thank Dr. Güzin Bayraksan (Ohio State University) and Dr. Fujun Lan (University of Maryland School of Medicine) for their helpful discussions on stochastic optimization.

REFERENCES

- [1] M. J. Abdel-Rahman, F. Lan, and M. Krunz, “Spectrum-efficient stochastic channel assignment for opportunistic networks,” in *Proc. IEEE GLOBECOM Conf.*, Dec. 2013, pp. 1272–1277.
- [2] FCC, “Second memorandum opinion and order in the matter of unlicensed operation in the TV broadcast bands (ET Docket No. 04-186),” Additional Spectrum for Unlicensed Devices Below 900 MHz and in 3 GHz Band, Washington, DC, USA, (EC Docket No. 02-380), Sep. 23, 2010.
- [3] K. Harrison, S. M. Mishra, and A. Sahai, “How much white-space capacity is there?” in *Proc. IEEE DySPAN Conf.*, 2010, pp. 1–10.
- [4] Y. Wu, E. Pliszka, B. Caron, P. Bouchar, and G. Chouinard, “Comparison of terrestrial DTV transmission systems: The ATSC 8-VSB, the DVB-T COFDM, and the ISDB-T BST-OFDM,” *IEEE Trans. Broadcast.*, vol. 46, no. 2, pp. 101–113, Jun. 2000.
- [5] V. Angelakis, S. Papadakis, V. Siris, and A. Traganitis, “Adjacent channel interference in 802.11a is harmful: Testbed validation of a simple quantification model,” *IEEE Commun. Mag.*, vol. 49, no. 3, pp. 160–166, Mar. 2011.
- [6] A. Zubow and R. Sombrutzki, “Adjacent channel interference in IEEE 802.11n,” in *Proc. IEEE WCNC Conf.*, Apr. 2012, pp. 1163–1168.
- [7] D. Skordoulis et al., “IEEE 802.11n MAC frame aggregation mechanisms for next-generation high-throughput WLANs,” *IEEE Wireless Commun. Mag.*, vol. 15, no. 1, pp. 40–47, Feb. 2008.
- [8] L. Deek, E. Garcia-Villegas, E. Belding, S.-J. Lee, and K. Almeroth, “The impact of channel bonding on 802.11n network management,” in *Proc. ACM CoNEXT Conference*, Dec. 2011, p. 11.
- [9] L. Deek, E. Garcia-Villegas, E. Belding, S. Lee, and K. Almeroth, “Intelligent channel bonding in 802.11n WLANs,” *IEEE Trans. Mobile Comput.*, vol. 13, no. 6, pp. 1242–1255, Jun. 2014.
- [10] M. Park, “IEEE 802.11ac: Dynamic bandwidth channel access,” in *Proc. IEEE ICC Conf.*, Jun. 2011, pp. 1–5.
- [11] O. Bejarano, E. Knightly, and M. Park, “IEEE 802.11ac: From channelization to multi-user MIMO,” *IEEE Commun. Mag.*, vol. 51, no. 10, pp. 84–90, Oct. 2013.
- [12] G. Yuan, X. Zhang, W. Wang, and Y. Yang, “Carrier aggregation for LTE-advanced mobile communication systems,” *IEEE Commun. Mag.*, vol. 48, no. 2, pp. 88–93, Feb. 2010.
- [13] H. Zhu and J. Wang, “Chunk-based resource allocation in OFDMA systems—Part I: Chunk allocation,” *IEEE Trans. Commun.*, vol. 57, no. 9, pp. 2734–2744, Sep. 2009.
- [14] H. Zhu and J. Wang, “Chunk-based resource allocation in OFDMA systems—Part II: Joint chunk, power and bit allocation,” *IEEE Trans. Commun.*, vol. 60, no. 2, pp. 499–509, Sep. 2012.
- [15] L. Lei, V. Angelakis, and D. Yuan, “Performance analysis of chunk-based resource allocation in wireless OFDMA systems,” in *Proc. IEEE Int. Workshop CAMAD Commun. Links Netw.*, 2012, pp. 90–94.
- [16] I. Macaluso, L. DaSilva, and L. Doyle, “Learning Nash equilibria in distributed channel selection for frequency-agile radios,” in *Proc. ECAI Workshop Artif. Intell. Telecommun. Sensor Netw.*, Aug. 2012, pp. 7–10.
- [17] Y. Xiao, T. Forde, I. Macaluso, L. DaSilva, and L. Doyle, “Spatial spectrum sharing-based carrier aggregation for heterogeneous networks,” in *Proc. IEEE GLOBECOM Conf.*, 2012, pp. 2591–2596.
- [18] Y. Xiao, C. Yuen, P. Di Francesco, and L. DaSilva, “Dynamic spectrum scheduling for carrier aggregation: A game theoretic approach,” in *Proc. IEEE ICC Conf.*, 2013, pp. 2672–2676.
- [19] G. S. Uyanik, M. J. Abdel-Rahman, and M. Krunz, “Optimal guard-band-aware channel assignment with bonding and aggregation in multi-channel systems,” in *Proc. IEEE GLOBECOM Conf.*, Dec. 2013, pp. 4898–4903.
- [20] G. S. Uyanik, M. J. Abdel-Rahman, and M. Krunz, “Optimal channel assignment with aggregation in multi-channel systems: A resilient approach to adjacent-channel interference,” *Ad Hoc Netw. J.*, vol. 20, pp. 64–76, Sep. 2014.
- [21] S. Martello and P. Toth, *Knapsack Problems: Algorithms and Computer Implementations*. Hoboken, NJ, USA: Wiley, 1990.
- [22] P. Kall and S. W. Wallace, *Stochastic Programming*. Hoboken, NJ, USA: Wiley, 1994.
- [23] A. Klappenecker, “Markov’s Inequality.” [Online]. Available: <http://faculty.cs.tamu.edu/klappi/csce689-s10/markov.pdf>.
- [24] A. Muqattash and M. Krunz, “POWMAC: A single-channel power-control protocol for throughput enhancement in wireless ad hoc networks,” *IEEE J. Sel. Areas Commun.*, vol. 23, no. 5, pp. 1067–1084, May 2005.
- [25] H. Bany Salameh and M. Krunz, “Adaptive power-controlled MAC protocols for improved throughput in hardware-constrained cognitive radio networks,” *Ad Hoc Netw. J.*, vol. 9, no. 7, pp. 1127–1139, Sep. 2011.
- [26] T. H. Cormen, C. Stein, R. L. Rivest, and C. E. Leiserson, *Introduction to Algorithms*, 2nd ed. New York, NY, USA: McGraw-Hill, 2001.



Mohammad J. Abdel-Rahman (S’12–M’15) received the Ph.D. degree in electrical and computer engineering from the University of Arizona in 2014. He is a Postdoctoral Associate in the Electrical and Computer Engineering Department at Virginia Polytechnic Institute and State University. His research interests include the areas of wireless communications and networking, with emphasis on dynamic spectrum access networks, wireless security, resource management, adaptive protocols, satellite communications, and wireless sensor networks. He serves as a reviewer for several international conferences and journals.



Marwan Krunz (S’93–M’95–SM’04–F’10) received the Ph.D. degree in electrical engineering from Michigan State University in 1995. He is a Professor of ECE and CS at the University of Arizona. He is the Site Codirector at the US National Science Foundation (NSF) Broadband Wireless Access and Applications Center. He joined the University of Arizona in January 1997, after a brief postdoctoral stint at the University of Maryland. In 2010, he was a Visiting Chair of Excellence at the University of Carlos III de Madrid. He previously held other visiting research positions at INRIA-Sophia Antipolis, HP Labs, University of Paris VI, University of Paris V, and US West Advanced Technologies. His research interests include the areas of wireless communications and networking, with emphasis on resource management, adaptive protocols, and security issues. He has published more than 215 journal articles and peer-reviewed conference papers, and is a coinventor on five US patents. He received the 2012 IEEE TCCC Outstanding Service Award. He received the US NSF CAREER Award in 1998. He currently serves on the editorial board for the IEEE TRANSACTIONS ON COGNITIVE COMMUNICATIONS AND NETWORKING. Previously, he served on the editorial boards for the IEEE/ACM TRANSACTIONS ON NETWORKING, IEEE TRANSACTIONS ON MOBILE COMPUTING, IEEE TRANSACTIONS ON NETWORK AND SERVICE MANAGEMENT, *Computer Communications Journal*, and the *IEEE Communications Interactive Magazine*. He was the General Cochair for WiSec’12, and served as a TPC Chair for INFOCOM’04, SECON’05, and WoWMoM’06. He was the keynote speaker, an invited panelist, and a tutorial presenter at numerous international conferences. He is an Arizona Engineering Faculty fellow (2011–2014), and a IEEE Communications Society Distinguished lecturer (2013 and 2014).