Power Minimization in MIMO Cognitive Networks using Beamforming Games

Diep N. Nguyen and Marwan Krunz
Department of Electrical and Computer Engineering, University of Arizona
E-mail:{dnnguyen, krunz}@email.arizona.edu

***Abstract*—We consider a multi-channel multi-user cognitive radio MIMO network in which each node controls its antenna radiation directions and allocates power for each data stream by adjusting its *precoding matrices*. Under a noncooperative game, we optimize the set of precoding matrices (one per channel) at each node so as to minimize the total transmit power in the network. Using recession analysis and the theory of variational inequalities, we obtain sufficient conditions that guarantee the existence and uniqueness of the game's Nash Equilibrium (NE). Low-complexity distributed algorithms are also developed by exploiting the strong duality of the convex per-user optimization problem. To improve the efficiency of the NE, we introduce pricing policies that employ a novel network interference function. Existence and uniqueness of the new NE under pricing are studied. Simulations confirm the effectiveness of our joint optimization approach.**

***Index Terms*—Power/spectrum efficiency, beamforming, noncooperative game, pricing, cognitive radio, MIMO.**

I. Introduction

Cognitive radios (CRs) improve spectrum utilization by exploiting temporarily idle frequency bands. The spectral efficiency can be further boosted if CR nodes are equipped with multiple antennas to leverage communications in the space dimension through multi-input multi-output (MIMO) communications. To be more spectrally efficient, available channels can be shared among several CR links, so that a channel can be occupied by more than one link at the same time.

Besides the spatial multiplexing gain, a multi-antenna transmitter can also adjust its radiation pattern (beamforming). This feature is particularly helpful in managing network interference in a multi-user setting. MIMO transmitters can configure their radiation patterns to keep interference away from unintended receivers. Consequently, network throughput (or the power required to meet the rate demands) can be improved (or saved). The power allocation over multiplexed data streams as well as the beam pattern of a MIMO node can be jointly controlled by tuning the phases and amplitudes of the complex elements of its *precoding matrices* [1]. In this paper, we design an optimal set of precoding matrices (one per channel) for each CR MIMO node that allocate power over *both* space/antenna and frequency dimensions and yield optimal radiation patterns, so that the total transmit power is minimized subject to given rate demands.

A. Related Works

Most existing works on MIMO ad hoc networks (e.g., [2] [3] [4] [5]) often overlook the spectrum management aspect and do not optimize over the frequency dimension. Even without beamforming, joint optimization of spectrum and power over various data streams and channels is challenging. The network-wide joint power and spectrum allocation problem of a single-antenna network was recently shown to be NP-hard [6]. In fact, the number of variables in a MIMO dynamic spectrum sharing network grows quadratically with the number of antennas. Hence, if we were to rely on suboptimal solutions, the large number of variables involved makes it computationally expensive, even for centralized implementation. Other centralized approaches employing the Nash Bargaining schemes (e.g., [7] [8]) are also not applicable, as they require global network information. Given these facts, distributed solutions have been sought under the framework of noncooperative games [9]. In such a setup, nodes/links act individually to maximize their rates (referred to as the rate maximization game) or minimize the power required to meet given QoS/rate constraints (referred to as the power minimization game).

Unlike the rate maximization (RM) game where players' strategic spaces are independent (e.g., [2] [3] [10] [11] [12] [13] [14]), the power minimization (PM) game exhibits complex coupling between these strategic spaces. Specifically, the strategic space of a link in the RM game is defined by its available resources, e.g., power, available channels, antennas, etc., which do not depend on other players' actions. In contrast, in a PM problem with rate constraints, the strategic space of a player is not only shaped by its resources but also by its achievable rate, which is a function of the actions of all other players. For example, it can be proved that the RM game always admits a NE [10] [12]. In contrast, the PM game may not have a NE (e.g., the rate demands exceed the network capacity). Moreover, under power budget constraints, the strategic space of a player under the PM game can be empty (e.g., when the power budget is not sufficient to support the rate demand, given the interference from other transmitters). In the context of a CR network, the dynamics of primary users (PUs) also affect if a requested rate can be met or not. Existing PM works tend to overlook these facts (e.g., [15] [16] [17] [18] [19]), and often assume the existence of a NE. While the projection method [20] and the variational inequality theory [2] [11] [21] have been instrumental in tackling the RM game, these techniques require nonempty strategic spaces.

Recently, the PM game for SISO networks was tackled [22], where conditions for the existence and uniqueness of the NE were established. The application of the methodology in [22] to MIMO networks with dynamic spectrum sharing and beamforming is not trivial. First, despite the presence of complex coupling among the strategic spaces, SISO networks

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without beamforming are still amenable to an explicit relationship between the power allocation from different users and the achievable rate of a given user. This was critical in analyzing the existence and uniqueness of the NE in [22]. In MIMO systems, such a relationship is implicit, as power allocation is carried out through matrix manipulations. Second, the achievable rate of a link depends not only on how much power is allocated to that link and its interferers, but also on how they allocate power over multiple data streams in both space and frequency dimensions. In the SISO case without beamforming, the power is radiated equally in all directions. However, using beamforming with multiple antennas, the optimal radiation directions of a transmitter depends on its channel gain matrices, other links' channel gain matrices, and their antenna patterns. Moreover, the existence and uniqueness of the NE in a MIMO game with pricing has not been studied.

B. Contributions

In this paper, we model the joint problem of power assignment, spectrum allocation, and beamforming as a noncooperative game. We derive conditions for the existence and uniqueness of the game's NE. Intuitively, these conditions are met if the power budget is sufficient enough to satisfy the rate demands, the requested rates are not too high to harm PU receptions, the PUs' interference to CRs is not too strong, and the CR interference is not too severe. The four conditions are quantified in a way that allows a node to instantly decide its appropriate rate.

Our second contribution is in deriving user-dependent pricing policies that significantly improve the NE's efficiency. At each transmitter, the pricing function uses a diagonal block pricing-factor matrix to capture the interference effect from this transmitter to unintended receivers. Intuitively, the pricing function guides a transmitter to steer its radiation directions away from unintended receivers. The existence and uniqueness of the NE under pricing are also investigated.

Third, exploiting the strong duality in convex optimization, we design a low-complexity distributed algorithm to determine the set of precoding matrices (*best reponse*) for each node. The number of variables in the distributed algorithm is $K+2$, where K is the number of frequency bands (hence, the algorithm is independent of the antenna array size). Simulations show that the distributed algorithm converges to the unique NE under both synchronous and asynchronous updates.

Our setup uses full/generalized eigen MIMO precoders. This differs from a large body of works on MIMO precoder design (e.g., [15] [23] [24]), where only one data stream is sent from a MIMO transmitter. In these works, precoders are of rank of one. In generalized eigencoding, there is no constraint on the rank of the precoding matrices [16], i.e., several data streams can be sent simultaneously.

Throughout the paper, $(.)^*$ denotes the conjugate of a matrix, $(.)^H$ denotes its Hermitian transpose, tr$(.)$ denotes its trace, $|.|$ denotes its determinant, $||.||$ denotes the Euclidean (or Frobenius) norm, $(.)^T$ denotes the matrix transpose. $\text{eig}_{\max}(.)$, $\text{eig}_{\min}(.)$, and $\text{diag}_s(.)$ indicate the maximum, minimum eigenvalue, and the diagonal element (s,s) of a matrix, respectively. sum$(.)$ gives the summation of all elements of the vector. Matrices and vectors are bold-faced.

II. Problem Statement

A. System Model and the Network-wide Problem

We consider a multi-channel cognitive MIMO network that coexists with several PU networks in a rich-scattering environment (to facilitate MIMO spatial multiplexing). The network consists of N transmitter-receiver pairs (links). Each CR node is equipped with M antennas. The spectrum to be allocated is comprised of K orthogonal bands (referred to as channels or sub-carriers in OFDM) that have central frequencies f_1, f_2, …, f_K. Let $\Phi_N \stackrel{\text{def}}{=} \{1,2,\dots,N\}$ and $\Psi_K \stackrel{\text{def}}{=} \{1,2,\dots,K\}$ denote the sets of CR links and channels, respectively. Each CR i can simultaneously communicate over multiple frequencies (e.g., using non-contiguous OFDM). We impose a half-duplex constraint on all transmissions.

The transmitter of each CR link can send up to M independent data streams over each channel. Let $\mathbf{x}_u^{(k)}$ be an $M \times 1$ column vector, consisting of M information symbols (from M data streams), sent on link u using the channel with central frequency f_k (hereon also referred to as channel f_k for short). The radiation pattern and power allocation for the M streams of link u on channel f_k are determined by its precoding matrix $\tilde{\mathbf{T}}_u^{(k)}$. The actual transmit vector on channel f_k at the radio interface is $\tilde{\mathbf{T}}_u^{(k)}\mathbf{x}_u^{(k)}$.

We allow for spectrum sharing among various CR links. Specifically, for channel f_k, the signal vector $\mathbf{y}_u^{(k)}$ at the receiver of link u is given by:

$$\mathbf{y}_u^{(k)} = \mathbf{H}_{u,u}^{(k)}\tilde{\mathbf{T}}_u^{(k)}\mathbf{x}_u^{(k)} + \sum_{j\in\Phi_N\setminus\{u\}} \mathbf{H}_{u,j}^{(k)}\tilde{\mathbf{T}}_j^{(k)}\mathbf{x}_j^{(k)} + \mathbf{N}_k \qquad (1)$$

where $\mathbf{H}_{u,u}^{(k)}$ is a $M \times M$ channel gain matrix on channel f_k of link u. Each element of $\mathbf{H}_{u,u}^{(k)}$ is a multiplication of a distance- and channel-dependent attenuation term, and a complex Gaussian variable (with zero mean and unit variance) that reflects multi-path fading. $\mathbf{H}_{u,j}^{(k)}$ denotes the cross-channel gain matrix from the transmitter of link j to the unintended receiver of link u, $u \neq j$. The second term in (1) represents interference from transmitters of CR links $j \neq u$ that share channel f_k with link u. $\mathbf{N}_k$ is an $M\times 1$ complex Gaussian noise vector with covariance matrix $\mathbf{I}_k = (1+I_{pu}(k))\mathbf{I}$, representing the floor noise with unit variance plus (whitened) interference $I_{pu}(k)$ from PUs on channel f_k.

We assume that interference cancellation is not used. A receiver decodes its data streams by treating interference from other transmitters as colored noise. The Shannon rate over link u on channel f_k is [1]:

$$R_u^{(k)} = \log|\mathbf{I} + \tilde{\mathbf{T}}_u^{(k)H}\mathbf{H}_{u,u}^{(k)H}{\mathbf{C}_u^{(k)}}^{-1}\mathbf{H}_{u,u}^{(k)}\tilde{\mathbf{T}}_u^{(k)}| \qquad (2)$$

where $\mathbf{C}_u^{(k)}$ is the noise-plus-interference covariance matrix at the receiver of link u over channel f_k:

$$\mathbf{C}_u^{(k)} = (1+I_{pu}(k))\mathbf{I} + \sum_{j\in\Phi_N\setminus\{u\}} \mathbf{H}_{u,j}^{(k)}\tilde{\mathbf{T}}_j^{(k)}\tilde{\mathbf{T}}_j^{(k)H}\mathbf{H}_{u,j}^{(k)H}.$$

The total channel rate over all frequencies of link u is:

$$R_u = \sum_{k\in\Psi_K} R_u^{(k)}. \qquad (3)$$

PU protection is provided in the form of database-authorized access and frequency-dependent power masks on CR transmit powers. Note that the FCC [25] recently imposed power masks even for idle channels, if such channels are adjacent to PU-occupied channels (e.g., this power mask is 40 mW for bands adjacent to active TV bands). Let $\mathbf{P}_{mask} \stackrel{\text{def}}{=} (P_{\text{mask}}(f_1), P_{\text{mask}}(f_2), \ldots, P_{\text{mask}}(f_K))$ denote the power mask on various channels, we require:

$$\sum_{s=1}^{M} P_{s,k}^{(u)} = \text{tr}(\tilde{\mathbf{T}}_u^{(k)} \tilde{\mathbf{T}}_u^{(k)H}) \leq P_{\text{mask}}(f_k). \quad (4)$$

where $P_{s,k}^{(u)}$ denotes the power allocated on channel f_k (frequency dimension) over antenna s (space dimension) for the transmitter of link u.

The network-wide power minimization problem under rate demand c_u can be stated as follows:

$$\begin{aligned} &\underset{\{\tilde{\mathbf{T}}_u^{(k)}, \forall k \in \Psi_K, \forall u \in \Phi_N\}}{\text{minimize}} \sum_{u \in \Phi_N} \sum_{k \in \Psi_K} \text{tr}(\tilde{\mathbf{T}}_u^{(k)} \tilde{\mathbf{T}}_u^{(k)H}) \\ \text{s.t.} \quad & \text{C1: } c_u \leq R_u, \quad \forall u \in \Phi_N \\ & \text{C2: } \text{tr}(\tilde{\mathbf{T}}_u^{(k)} \tilde{\mathbf{T}}_u^{(k)H}) \leq P_{\text{mask}}(f_k), \quad \forall k \in \Psi_K, \forall u \in \Phi_N. \\ & \text{C3: } \sum_{k \in \Psi_K} \text{tr}(\tilde{\mathbf{T}}_i^{(k)} \tilde{\mathbf{T}}_i^{(k)H}) \leq P_{\max}. \end{aligned} \quad (5)$$

B. Game Theoretic Design

The network optimization problem (5) is not convex and known to be NP-hard [6]. For a transmitter to compute its optimal set of precoders in a *distributed* manner with reasonable complexity, we formulate (5) as a strategic noncooperative game where the players are the N CR links. These players aim at maximizing their utilities, defined as the negative of their power consumption. The game's strategic space is the union of the strategic spaces of various players, subject to constraints C1, C2, C3 in (5). Each player/link u competes against others by selecting his strategic action of K precoders, denoted by $\tilde{\mathbf{T}}_u \stackrel{\text{def}}{=} (\tilde{\mathbf{T}}_u^{(1)}, \tilde{\mathbf{T}}_u^{(2)}, \ldots, \tilde{\mathbf{T}}_u^{(K)})$. $\tilde{\mathbf{T}}_u$ is an $M \times KM$ block matrix, comprised of K $M \times M$ matrices.

The payoff for player u, given below, is a function of its action $\tilde{\mathbf{T}}_u$ as well as other players' actions, $\tilde{\mathbf{T}}_{-u} \stackrel{\text{def}}{=} (\tilde{\mathbf{T}}_1, \tilde{\mathbf{T}}_2, \ldots, \tilde{\mathbf{T}}_{u-1}, \tilde{\mathbf{T}}_{u+1}, \ldots, \tilde{\mathbf{T}}_N)$:

$$U_u(\tilde{\mathbf{T}}_u, \tilde{\mathbf{T}}_{-u}) \stackrel{\text{def}}{=} -\sum_{k \in \Psi_K} \text{tr}(\tilde{\mathbf{T}}_u^{(k)} \tilde{\mathbf{T}}_u^{(k)H}). \quad (6)$$

The transmitter of each link assigns its power over both the space and frequency dimensions, and configures its radiation pattern to maximize its own return. Formally, each CR user u solves the following problem for its optimal precoders $\tilde{\mathbf{T}}_u$:

$$\begin{aligned} &\underset{\{\tilde{\mathbf{T}}_u^{(k)}, \forall k \in \Psi_K\}}{\text{maximize}} \quad U_u(\tilde{\mathbf{T}}_u, \tilde{\mathbf{T}}_{-u}) \\ \text{s.t.} \quad & \text{C1': } R_u \geq c_u \\ & \text{C2': } \text{tr}(\tilde{\mathbf{T}}_u^{(k)} \tilde{\mathbf{T}}_u^{(k)H}) \leq P_{\text{mask}}(f_k), \quad \forall k \in \Psi_K \\ & \text{C3': } \sum_{k \in \Psi_K} \text{tr}(\tilde{\mathbf{T}}_u^{(k)} \tilde{\mathbf{T}}_u^{(k)H}) \leq P_{\max}. \end{aligned} \quad (7)$$

III. Existence and Uniqueness of the NE

Intuitively, three factors affect the existence of a NE of (7): Network (multi-user) interference, PU protection requirement (through power masks), and nodes' power budget. To deal with the network interference, we first remove the power mask and power budget constraints (these constraints will be incorporated later) and have the following problem:

$$\begin{aligned} &\underset{\{\tilde{\mathbf{T}}_u^{(k)} \forall k \in \Psi_K\}}{\text{minimize}} \quad \sum_{k \in \Psi_K} \text{tr}(\tilde{\mathbf{T}}_u^{(k)} \mathbf{I} \tilde{\mathbf{T}}_u^{(k)H}) \\ & \text{s.t. C1' as in problem (7).} \end{aligned} \quad (8)$$

We rewrite the precoding matrix $\tilde{\mathbf{T}}_u^{(k)}$ as follows:

$$\tilde{\mathbf{T}}_u^{(k)} = \mathbf{T}_u^{(k)} \times \mathbf{P}_k^{(u)^{1/2}} \quad (9)$$

where $\mathbf{T}_u^{(k)}$ is an $M \times M$ matrix with unit-norm column vectors, specifying the directions to which the antenna array of node u points its beams. $\mathbf{P}_k^{(u)}$ is an $M \times M$ diagonal matrix whose entry (s, s) is the power allocated for sub-channel (s, k), $P_{s,k}^{(u)}$. Both $\mathbf{T}_u^{(k)}$ and $\mathbf{P}_k^{(u)}$ shape the antenna patterns.

At a NE, let $\boldsymbol{p}_u^{(k)} \stackrel{\text{def}}{=} (P_{1,k}^{(u)}, P_{2,k}^{(u)}, \ldots, P_{M,k}^{(u)})$ be a $1 \times M$ nonnegative vector, which denotes the power allocation vector of link u on M antennas at frequency f_k. Let $\boldsymbol{p}_u \stackrel{\text{def}}{=} (\boldsymbol{p}_u^{(1)}, \boldsymbol{p}_u^{(2)}, \ldots, \boldsymbol{p}_u^{(K)})$ be a $1 \times MK$ vector, which denotes the power allocation on all antennas and frequencies of link u. Let $\boldsymbol{p} \stackrel{\text{def}}{=} (\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots, \boldsymbol{p}_N) \in \mathbb{R}_+^{NKM}$ denote the power allocation on all antennas and frequencies of all players.

We observe that the unit matrix $\mathbf{I}$ is positive definite, so the objective function in (8) is non-decreasing in every element of $\boldsymbol{p}_u$. In other words, at a NE of the game (if one exists), the inequality constraint C1' becomes equality. Otherwise, one can still lower the power consumption to achieve a smaller value for the objective function while meeting the rate demand. This fact defines a feasible set for $\boldsymbol{p}$, denoted by $\mathbb{Q}_{feasible}(\boldsymbol{c})$, corresponding to a given requested rate profile $\boldsymbol{c} \stackrel{\text{def}}{=} (c_1, c_2, \ldots, c_N)$ at a NE. For a given rate profile $\boldsymbol{c}$, the game (8) has at least one bounded NE and only bounded NEs, if $\mathbb{Q}_{feasible}(\boldsymbol{c})$ is nonempty and bounded.

Theorem 1: Let $\mathbf{G}_k$ be defined in (11). If $\boldsymbol{G_k}$ is a P-matrix[1] $\forall k \in \Psi_K$, then $\mathbb{Q}_{feasible}(\boldsymbol{c})$ contains at least one bounded vector $\boldsymbol{p} \in \mathbb{R}_+^{NKM}$ and only bounded vectors $\boldsymbol{p}$. In other words, the game (8) admits at least one bounded NE and only bounded NEs.

Proof: We first claim that $\mathbb{Q}_{feasible}(\boldsymbol{c})$ contains at least one bounded vector $\boldsymbol{p} \in \mathbb{R}_+^{NKM}$ or the existence of a bounded NE to the game (8):

Lemma 1: Given that $\boldsymbol{G_k}$ is a P-matrix $\forall k \in \Psi_K$, then there exists at least one bounded vector $\boldsymbol{p} \in \mathbb{Q}_{feasible}(\boldsymbol{c}) \in \mathbb{R}_+^{NKM}$.
Proof: See [27]. □

The remaining task is to show that the game (8) admits only bounded NEs or $\mathbb{Q}_{feasible}(\boldsymbol{c})$ is bounded. For that, we rely on the concept of asymptotic cone of a nonempty set in recession analysis [28], as follows.

For a nonempty set $\mathbb{Q} \in \mathbb{R}_+^N$, its *asymptotic cone*, denoted by $\mathbb{Q}_{asymp}$, consists of vectors $\boldsymbol{d} \in \mathbb{R}_+^N$, referred to as *limit directions*. Each limit direction vector $\boldsymbol{d}$ is defined through the existence of a sequence of vectors $\boldsymbol{p}_n \in \mathbb{Q}$ and a sequence of scalars ν_n tending to $+\infty$ such that [28]:

$$\lim_{n \to \infty} \frac{\boldsymbol{p}_n}{\nu_n} = \boldsymbol{d}. \quad (14)$$

[1] A matrix is a P-matrix if all of its principal minors are positive [26].

$$\mathbb{Q}_{feasible}(\boldsymbol{c}) \overset{\text{def}}{=} \left\{ \boldsymbol{p} \in \mathbb{R}_+^{NKM} | R_u(\boldsymbol{p}) \overset{\text{def}}{=} \sum_{k \in \Psi_K} \log |\mathbf{I} + \tilde{\mathbf{T}}_u^{(k)H} \mathbf{H}_{u,u}^{(k)H} {\mathbf{C}_u^{(k)}}^{-1} \mathbf{H}_{u,u}^{(k)} \tilde{\mathbf{T}}_u^{(k)}| = c_u, \forall u \in \Phi_N \right\} \tag{10}$$

$$\mathbf{G}_k \overset{\text{def}}{=} \begin{bmatrix} |\mathbf{H}_{11}^{(k)H} \mathbf{H}_{1,1}^{(k)}|^{\frac{1}{M}} & -(2^{c_1}-1)\frac{\text{tr}(\mathbf{H}_{1,2}^{(k)H}\mathbf{H}_{1,2}^{(k)})}{M} & \cdots & -(2^{c_1}-1)\frac{\text{tr}(\mathbf{H}_{1,N}^{(k)H}\mathbf{H}_{1,N}^{(k)})}{M} \\ -(2^{c_2}-1)\frac{\text{tr}(\mathbf{H}_{2,1}^{(k)H}\mathbf{H}_{2,1}^{(k)})}{M} & |\mathbf{H}_{2,2}^{(k)H} \mathbf{H}_{2,2}^{(k)}|^{\frac{1}{M}} & \cdots & -(2^{c_2}-1)\frac{\text{tr}(\mathbf{H}_{2,N}^{(k)H}\mathbf{H}_{2,N}^{(k)})}{M} \\ \vdots & \vdots & \ddots & \vdots \\ -(2^{c_N}-1)\frac{\text{tr}(\mathbf{H}_{N,1}^{(k)H}\mathbf{H}_{N,1}^{(k)})}{M} & -(2^{c_N}-1)\frac{\text{tr}(\mathbf{H}_{N,2}^{(k)H}\mathbf{H}_{N,2}^{(k)})}{M} & \cdots & |\mathbf{H}_{N,N}^{(k)H} \mathbf{H}_{N,N}^{(k)}|^{\frac{1}{M}} \end{bmatrix} \tag{11}$$

$$\mathbb{Q}_{asymp}(\boldsymbol{c}) \overset{\text{def}}{=} \left\{ \boldsymbol{d} \in \mathbb{R}_+^{NKM} | \exists \{\boldsymbol{p}_n\} \in \mathbb{Q}_{feasible}(\boldsymbol{c}) \text{ and } \{\nu_n\} \to +\infty \text{ so that } \lim_{n\to\infty} \frac{\boldsymbol{p}_n}{\nu_n} = \boldsymbol{d} \right\} \tag{12}$$

$$\mathbb{Q}(\boldsymbol{c}) \overset{\text{def}}{=} \left\{ \boldsymbol{d} \in \mathbb{R}_+^{NKM} | R'_u(\boldsymbol{d}) \overset{\text{def}}{=} \sum_{k \in \Psi_K} \log \left(1 + \frac{\text{tr}(\tilde{\mathbf{T}}_u^{(k)H}\tilde{\mathbf{T}}_u^{(k)}) |\mathbf{H}_{u,u}^{(k)H}\mathbf{H}_{u,u}^{(k)}|^{\frac{1}{M}}}{\sum\limits_{j \in \Phi_N \setminus \{u\}} \frac{\text{tr}(\mathbf{H}_{u,j}^{(k)H}\mathbf{H}_{u,j}^{(k)})}{M} \text{tr}(\tilde{\mathbf{T}}_j^{(k)H}\tilde{\mathbf{T}}_j^{(k)})} \right) \le c_u, \forall u \in \Phi_N \right\} \tag{13}$$

The set $\mathbb{Q}$ is bounded if its asymptotic cone $\mathbb{Q}_{asymp}$ contains only the zero vector $\mathbf{0}$ [28]. Applying this to the set $\mathbb{Q}_{feasible}(\boldsymbol{c})$, the game (8) admits only bounded NEs if its asymptotic cone $\mathbb{Q}_{asymp}(\boldsymbol{c})$ contains only the zero vector. The asymptotic cone $\mathbb{Q}_{asymp}(\boldsymbol{c})$ is formally defined in (12).

Given that $\mathbb{Q}_{feasible}(\boldsymbol{c})$ has at least one bounded $\boldsymbol{p}$ (Lemma 1), it is clear that the vector zero $\mathbf{0}$ belongs to its asymptotic cone $\mathbb{Q}_{asymp}(\boldsymbol{c})$ (by the definition of limit directions). We now construct a set $\mathbb{Q}(\boldsymbol{c})$ of which $\mathbb{Q}_{asymp}(\boldsymbol{c})$ is a subset and prove that $\mathbb{Q}(\boldsymbol{c}) = \{\mathbf{0}\}$ if $\boldsymbol{G_k}$ is a P-matrix $\forall k \in \Psi_K$.

Lemma 2: If $\boldsymbol{d} \in \mathbb{Q}_{asymp}(\boldsymbol{c})$ then $\boldsymbol{d}$ belongs to $\mathbb{Q}(\boldsymbol{c})$, defined in (13).

Proof: See [27]. □

Assuming that there exists at least one $\boldsymbol{d} \neq \mathbf{0}$ and that $\boldsymbol{d} \in \mathbb{Q}(\boldsymbol{c})$, then $\forall u \in \Phi_N$:

$$\log \left(1 + \frac{\text{tr}(\tilde{\mathbf{T}}_u^{(k)H}\tilde{\mathbf{T}}_u^{(k)}) |\mathbf{H}_{u,u}^{(k)H}\mathbf{H}_{u,u}^{(k)}|^{\frac{1}{M}}}{\sum\limits_{j \in \Phi_N \setminus \{u\}} \frac{\text{tr}(\mathbf{H}_{u,j}^{(k)H}\mathbf{H}_{u,j}^{(k)})}{M} \text{tr}(\tilde{\mathbf{T}}_j^{(k)H}\tilde{\mathbf{T}}_j^{(k)})} \right) \le c_u \tag{15a}$$

$$\text{tr}(\tilde{\mathbf{T}}_u^{(k)H}\tilde{\mathbf{T}}_u^{(k)}) |\mathbf{H}_{u,u}^{(k)H}\mathbf{H}_{u,u}^{(k)}|^{\frac{1}{M}} - \tag{15b}$$

$$(2^{c_u}-1) \sum_{j \in \Phi_N \setminus \{u\}} \text{tr}(\tilde{\mathbf{T}}_j^{(k)H}\tilde{\mathbf{T}}_j^{(k)}) \frac{\text{tr}(\mathbf{H}_{u,j}^{(k)H}\mathbf{H}_{u,j}^{(k)})}{M} \le 0 \tag{15c}$$

$$\mathbf{G}_k \times [\text{tr}(\tilde{\mathbf{T}}_1^{(k)H}\tilde{\mathbf{T}}_1^{(k)}), \ldots, \text{tr}(\tilde{\mathbf{T}}_N^{(k)H}\tilde{\mathbf{T}}_N^{(k)})]^T \le \mathbf{0}. \tag{15d}$$

As $\mathbf{G}_k$ is a P-matrix for all $k \in \Psi_K$ and $[\text{tr}(\tilde{\mathbf{T}}_1^{(k)H}\tilde{\mathbf{T}}_1^{(k)}), \ldots, \text{tr}(\tilde{\mathbf{T}}_u^{(k)H}\tilde{\mathbf{T}}_u^{(k)}), \ldots, \text{tr}(\tilde{\mathbf{T}}_N^{(k)H}\tilde{\mathbf{T}}_N^{(k)})]^T$ is a nonnegative vector, (15d) implies that $\text{tr}(\tilde{\mathbf{T}}_u^{(k)H}\tilde{\mathbf{T}}_u^{(k)}) = 0 \;\; \forall u \in \Phi_N$ and $\forall k \in \Psi_K$ [26] or $\boldsymbol{d} = \mathbf{0}$. This contradicts the above assumption. Hence, $\mathbb{Q}(\boldsymbol{c})$ and its subset $\mathbb{Q}_{asymp}(\boldsymbol{c})$ equal to $\{\mathbf{0}\}$. Theorem 1 is proved. □

We now give some intuitions behind Theorem 1. If the diagonal elements of $\mathbf{G}_k$ are positive, then a sufficient condition for $\mathbf{G}_k$ to be a P-matrix is $|\mathbf{G}_k(u,u)| \ge \sum\limits_{j \neq u} |\mathbf{G}_k(u,j)|$ (i.e., row diagonally dominant) [26]. Hence, the following inequality guarantees that game (8) has at least one bounded NE and only bounded NEs:

$$\frac{M \det(\mathbf{H}_{u,u}^{(k)H}\mathbf{H}_{u,u}^{(k)})^{\frac{1}{M}}}{\sum\limits_{j \in \Phi_N \setminus \{u\}} \text{tr}(\mathbf{H}_{u,j}^{(k)H}\mathbf{H}_{u,j}^{(k)})} \ge (2^{c_u}-1) \; \forall\, k \in \Psi_K, \forall \text{ u} \in \Phi_N. \tag{16}$$

The nominator of the LHS in (16) represents the strength of the channel gain matrix of link u on channel f_k, while its denominator describes the strength of cross-(interfering) channel gain matrices from other links j, $j \neq u$, on the receiver of link u. First, for the game (8) to have at least one NE (at which the required powers of all links are bounded), the multi-user interference in each channel f_k should not be too strong. Second, the acceptable multi-user interference is explicitly quantified in (16), and is a function of the rate demand c_u of each link u. For higher rate demands, inequality (16) becomes stringent, meaning that lower multi-user interference is necessary. Hence, inequality (16) can be used as a criterion to reject or admit a newly requested transmission/rate. When links set their target rate too high that inequality (16) does not hold, a bounded NE may not exist. In this case, nodes keep increasing their transmit powers to meet their rate demands. Network interference becomes more severe and no link reaches its requested rate (interference-limited communications).

To better interpret inequality (16), recall that each element of channel gain matrices in (16) is the product of a complex Gaussian variable with zero mean and unit variance (in the $\bar{\mathbf{H}}_{u,u}^{(k)}$ matrix) and the distance-dependence attenuation factor: $\mathbf{H}_{u,u}^{(k)} = \frac{1}{\sqrt{d_{u,u}^n}} \bar{\mathbf{H}}_{u,u}^{(k)}$ where n is the free-space attenuation factor and $d_{u,u}$ is the transmission distance of link u. Inequality (16) can be rewritten as:

$$\frac{M \det(\bar{\mathbf{H}}_{u,u}^{(k)H}\bar{\mathbf{H}}_{u,u}^{(k)})^{\frac{1}{M}}}{\sum\limits_{j \in \Phi_N \setminus \{u\}} \frac{d_{u,u}^n}{d_{u,j}^n} \text{tr}(\bar{\mathbf{H}}_{u,j}^{(k)H}\bar{\mathbf{H}}_{u,j}^{(k)})} \ge (2^{c_u}-1) \; \forall\, k \in \Psi_K, \forall \text{ u} \in \Phi_N. \tag{17}$$

(17) holds if the distance between the transmitter and the receiver is small enough compared with distances between the receiver and its interferers, the channel gain matrix of link u is full-rank (this is often the case in a rich-scattering environment)

and its requested rate is not too high.

Given the existence of bounded NEs to the game in (8), we now incorporate the power mask and power budget constraints in the following theorem.

Theorem 2: The game (7) admits at least one bounded NE and only bounded NEs if $\boldsymbol{G_k}$ is a P-matrix and the vector-inequality below holds element-by-element $\forall k \in \Psi_K$ and $\forall u \in \Phi_N$:

$$\mathbf{G}_k^{-1} \times \begin{bmatrix} 2^{c_1}-1 \\ 2^{c_2}-1 \\ \vdots \\ 2^{c_N}-1 \end{bmatrix} \leq \begin{bmatrix} \frac{P_{\text{mask}}(f_k)}{1+I_{pu}(k)} \\ \frac{P_{\text{mask}}(f_k)}{1+I_{pu}(k)} \\ \vdots \\ \frac{P_{\text{mask}}(f_k)}{1+I_{pu}(k)} \end{bmatrix} \text{ and } \text{sum}(\mathbf{P}_u) \leq P_{\max} \tag{18}$$

where

$$\boldsymbol{P}_u \stackrel{\text{def}}{=} \begin{bmatrix} (1+I_{pu}(1))\mathbf{G}_1^{-1}(u,:) \\ (1+I_{pu}(2))\mathbf{G}_2^{-1}(u,:) \\ \vdots \\ (1+I_{pu}(K))\mathbf{G}_K^{-1}(u,:) \end{bmatrix} \times \begin{bmatrix} (2^{c_1}-1) \\ (2^{c_2}-1) \\ \vdots \\ 2^{c_N}-1 \end{bmatrix}$$

and $\mathbf{G}_k^{-1}(u,:)$ is the uth row of the inverse of matrix ${\mathbf{G}_k}^2$.

Proof: See [27]. □

From (18), if PUs are more active on a given channel (higher $I_{pu}(k)$), the inequality becomes stricter. This means that CRs should reduce their transmission power on this channel to avoid interfering PUs. Moreover, as the inequality becomes tighter (smaller LHS of (18)) when PUs become more active, it is less likely for a NE to exist. Hence, besides the PU protection requirement, inequality (18) also shows the interference effect from PUs to CRs.

So far, we have derived conditions that capture the factors that affect the existence of a NE of the game (7). The conditions in Theorem 1 ensure that network interference is mild enough to support the requested rates. The conditions in the first inequality in Theorem 2 enforce that the requested rates are not too high to harm PUs reception given PUs' activities (indirectly captured by PUs' interference). The last inequality in Theorem 2 guarantees that rate demands are affordable given nodes' power budgets.

Interestingly, if one removes the resource and PUs protection constraints and sets the number of antenna to be one, the conditions in Theorem 1 reduce to the conditions derived for the NE existence in single-antenna (legacy) networks (in Theorem 5 of [22]). Moreover, the authors of [22] proved that their sufficient conditions become necessary when $K = 1$ and $M = 1$, i.e., a single-channel SISO network (Proposition 11 of [22]). They also showed that for the case $K = 1$ and $M = 1$, their sufficient conditions are identical to those in [29]. Hence, though we cannot show that the sufficient conditions in Theorems 1 and 2 are also necessary in general cases, the following corollary gives a sense of how tight the conditions in Theorem 1 are.

Corollary 1: If $M = 1$, the conditions in Theorem 1 become the sufficient for the NE existence derived for the SISO network in [22]. Furthermore, If $K = 1$, then the sufficient conditions for a NE existence in Theorem 1 become necessary and identical to those in [29].

[2] Since $\mathbf{G}_k$ is a P-matrix, it is invertible.

One may be curious about the relation between the NE existence and the fulfillment of rate demands. The following theorem shows that if the requested rates can be supported, then a NE must exist.

Theorem 3: If rate demands are supported, then the game (7) admits at least one NE.

Proof: See [27]. □

Theorem 3 also points out that a NE does not exist only if the requested rates are not met. In such a case, players whose rates are not achieved have to reduce their demands (or even leave the game to reduce network interference and facilitate other links' transmissions), and then repeat game (7). Investigating this process would require a repeated game, which is left for a future work.

To analyze the uniqueness of the NE, we resort to variational inequalities theory, casting (7) as a variational inequalities (VI) problem. A tutorial on the application of VI to communication systems can be found in [21].

Theorem 4: If game (7) has a NE, then this NE is unique.

Proof: We prove that the mapping of the equivalent VI problem of (7) is continuous uniformly-P function. Hence, if a NE exists, it is unique. See [27] for details. □

Theorem 4 indicates that (7) does not have multiple NEs. Hence, the NE existence condition of (7) is also the NE uniqueness condition (formally stated in Theorem 5 below).

Theorem 5: If the conditions in Theorem 2 hold, then there exists a unique NE of the game (7).

IV. Beamforming Game with Pricing

A. Pricing Policy Design

The social welfare of a noncooperative game can be improved with appropriate pricing/taxation policies [30] that make players more responsible for their actions. For our joint power allocation, spectrum management, and beamforming game, the utility function with price for link u is given by:

$$U_u^{'}(\tilde{\mathbf{T}}_u, \tilde{\mathbf{T}}_{-u}) \stackrel{\text{def}}{=} U_u(\tilde{\mathbf{T}}_u, \tilde{\mathbf{T}}_{-u}) - F_u(\tilde{\mathbf{T}}_u) \tag{19}$$

where $F(\tilde{\mathbf{T}}_u)$ is the pricing function for link u. The goal for player u is:

$$\begin{aligned} &\underset{\{\tilde{\mathbf{T}}_u^{(k)} \forall k \in \Psi_K\}}{\text{maximize}} \quad U_u^{'}(\tilde{\mathbf{T}}_u, \tilde{\mathbf{T}}_{-u}) \\ &\text{s.t. C1', C2', C3' as in problem (7).} \end{aligned} \tag{20}$$

Efficient pricing policies, which adapt to each individual player, can be designed by forcing the solution obtained from running per-user optimization problems in (20) to converge to a locally optimal solution of the network-wide (nonconvex) problem (5). This can be realized by using the K.K.T. conditions [31] to equate the stationary points of (20) to those of (5) (examples of this approach can be found in [12] and [10]). To ease the complexity of this procedure, the pricing functions are often linear [32]. However, in our case, following this procedure leads to a pricing function that depends on global information and the Lagrangian multipliers of (5) [33]. Such pricing policies are not suitable for distributed implementation.

The idea of characterizing and minimizing the total network interference of MIMO MANETs was first introduced in [15] for the case of beamforming, and then extended to generalized eigencoding in [16]. Network interference models in [15] and [16] are generalized forms of the total squared correlation function in CDMA systems [34]. However, the models in [15] and [16] are implicitly developed for full-duplex devices (the precoders are found to satisfy the data rate requirement in both directions). From a game theoretic point, precoders in [15] and [16] are obtained by introducing a pricing function that depends on the noise-plus-interference covariance matrices at both ends of the link. This approach is not applicable to half-duplex networks. Via simulations, we find that this approach is unstable and power-inefficient when applied to half-duplex MIMO transceivers.

In this work, we propose to quantify the network interference by the trace of all interference-plus-noise covariance matrices at all receivers. We refer to this trace as the Network Interference Function (NIF):

$$\begin{aligned}\text{NIF} &\stackrel{\text{def}}{=} \text{tr}\{\sum_{u\in\Phi_N}\sum_{k\in\Psi_K}\mathbf{C}_u^{(k)}\}\\ &= KN(1+I_{pu}(k))\text{tr}(\mathbf{I}) + \sum_{u\in\Phi_N}\text{tr}\left[\tilde{\mathbf{T}}_u^H\times\mathbf{A}_u\times\tilde{\mathbf{T}}_u\right].\end{aligned} \tag{21}$$

where $\mathbf{A}_u$ is a $KM\times KM$ block diagonal matrix. The kth block $\mathbf{A}_u^{(k)} = \sum_{i\in\Phi_N\setminus\{u\}}\mathbf{H}_{i,u}^{(k)H}\mathbf{H}_{i,u}^{(k)}$ is an $M\times M$ positive-semidefinite matrix.

NIF has the same physical unit as power (Watt). For single-antenna case, NIF is nothing but the total interference that transmitters induce on unintended receivers. However, in a SISO network, as transmissions are omnidirectional, we do not have the freedom to configure the radiation pattern to minimize NIF. For MIMO transmitters, first recall that the trace of a matrix is the sum of its eigenvalues. Additionally, for the existence of a NE, inequality (16) partially states the upper bound on the total channel gains from interferers, regardless of the transmit powers from these transmitters. Hence, NIF captures the *effective* interference from a transmitter to its unintended receivers for a given selection of precoders.

From (21), we observe that if each transmitter avoids as much as possible interfering with other receivers, captured by the trace of interference-plus-noise covariance matrices, NIF is then minimized. Intuitively, the transmitter can realize that by selecting its precoders such that the antenna's radiation beams are kept away as much as possible from unintended receivers. Hence, we now propose the following pricing function for link u:

$$F(\tilde{\mathbf{T}}_u) = \text{tr}\left[\tilde{\mathbf{T}}_u^H\mathbf{A}_u\tilde{\mathbf{T}}_u\right]. \tag{22}$$

$\mathbf{A}_u$ is referred to as the pricing-factor matrix of CR link u and $\mathbf{A}_u^{(k)}$ is referred to as the pricing-factor submatrix at channel f_k of link u. The per-user optimization problem (20) at transmitter u becomes:

$$\underset{\{\tilde{\mathbf{T}}_u^{(k)}\forall k\in\Psi_K\}}{\text{maximize}} \left\{-\sum_{k\in\Psi_K}\text{tr}(\tilde{\mathbf{T}}_u^{(k)}[\mathbf{I}+\mathbf{A}_u^{(k)}]\tilde{\mathbf{T}}_u^{(k)H})\right\} \tag{23}$$
s.t. C1', C2', C3' as in problem (7).

Note that to obtain its pricing function, a transmitter needs to know the channel matrix from itself to other receivers in its neighborhood. This information can be obtained by overhearing signalling packets at the MAC layer.

B. NE Characterization under Pricing

Because matrix $\mathbf{A}_u^{(k)}$ is positive semi-definite, $\mathbf{I}+\mathbf{A}_u^{(k)}$ is positive definite. In addition, the strategic space Q of the game (23) is identical to that of the game (7) (without pricing). One can apply the analysis of the game (7) and obtain the same conditions as in Theorem 2 for the NE existence of the game (23). The remaining task is to investigate the uniqueness of this NE. Following a similar procedure to the one in [35], one can map the game (23) to a $\text{VI}(Q,\bar{F})$ problem with:

$$\bar{F}_u \stackrel{\text{def}}{=} -\nabla U_u^{'}(\tilde{\mathbf{T}}_u,\tilde{\mathbf{T}}_{-u}) = [(\mathbf{A}_u^{(1)}+\mathbf{I})\tilde{\mathbf{T}}_u^{(1)},\ldots,(\mathbf{A}_u^{(K)}+\mathbf{I})\tilde{\mathbf{T}}_u^{(K)}]. \tag{24}$$

Let $\tilde{\mathbf{T}} \stackrel{\text{def}}{=} [\tilde{\mathbf{T}}_1\times\ldots\times\tilde{\mathbf{T}}_N]$ and $\tilde{\mathbf{T}}' \stackrel{\text{def}}{=} [\tilde{\mathbf{T}}_1'\times\ldots\times\tilde{\mathbf{T}}_N']$ be two different strategy sets of the strategic space of the game (23). We have:

$$\text{vec}(\tilde{\mathbf{T}}_u-\tilde{\mathbf{T}}_u')^T\text{vec}(\bar{F}(\tilde{\mathbf{T}}_u)-\bar{F}(\tilde{\mathbf{T}}_u'))$$

$$\geq \sum_{k\in\Psi_K}\text{eig}_{\min}(\mathbf{A}_u^{(k)}+\mathbf{I})||\text{vec}((\tilde{\mathbf{T}}_u^{(k)}-\tilde{\mathbf{T}}_u'^{(k)}))||^2 \tag{25a}$$

$$\geq \alpha||\text{vec}((\tilde{\mathbf{T}}_u-\tilde{\mathbf{T}}_u'))||^2 \tag{25b}$$

where vec is defined in (29) and $\alpha \stackrel{\text{def}}{=} \min_k\left\{\text{eig}_{\min}(\mathbf{A}_u^{(k)}+\mathbf{I})\right\}$. We use the fact that $||\mathbf{A}\boldsymbol{a}||\leq\text{eig}_{\max}(\mathbf{A})||\boldsymbol{a}||$ and the triangular inequality in (25a) and (25b), respectively.

Since $\mathbf{A}_u^{(k)}+\mathbf{I}$ is positive definite, $\text{eig}_{\min}(\mathbf{A}_u^{(k)}+\mathbf{I})>0$, meaning that the $\text{VI}(Q,\bar{F})$ problem or the game (23) has a unique NE. The following theorem summarizes our findings.

Theorem 6: The conditions in Theorem 2 guarantee that the beamforming game with pricing (23) has a unique NE.

V. Best Response and Distributed Algorithm

A. Optimal Radiation Directions and Power Allocation

We now solve the individual utility optimization problems (7) (without pricing) and (23) (with pricing). From these solutions, user u can determine its best response $\tilde{\mathbf{T}}_u = BR_u(\tilde{\mathbf{T}}_{-u})$. A solution to (7) can be obtained from that of (23) by setting the pricing-factor matrix to zero. Hence, we focus on solving (23).

Observe that the number of variables in problem (23) (herein referred to as the *primal problem*) is much greater than that in its dual problem ($2KM^2$ compared with $K+2$). Additionally, recalling the convexity of (23) and that the Slater's conditions can easily be shown to hold [31], strong duality holds for problem (23). A great deal of computational complexity can be saved by solving the dual problem of (23), instead of solving the primal one if the derivation of a closed-form dual function is not cumbersome [31]. Fortunately, the problem (23) possesses a special structure, which enables us to apply the Hadamard inequality to analytically derive its dual function. The dual problem of (23) is as follows:

$$\text{DP: } \underset{\{\boldsymbol{\lambda}_u\}}{\text{minimize}}\ D(\boldsymbol{\lambda}_u) \tag{30}$$

where $\boldsymbol{\lambda}_u \stackrel{\text{def}}{=} (\lambda_u^{(0)},\lambda_u^{(1)},\ldots,\lambda_u^{(K+1)})$ is a $1\times(K+1)$ vector of dual variables (the Lagrangian multipliers of the primal problem

$$L_u(\tilde{\mathbf{T}}_u,\boldsymbol{\lambda}_u) = -\sum_{k\in\Psi_K}\text{tr}(\tilde{\mathbf{T}}_u^{(k)H}[\mathbf{I}+\mathbf{A}_u^{(k)}]\tilde{\mathbf{T}}_u^{(k)}) - \lambda_u^{(0)}(c_u - R_u) - \sum_{k\in\Psi_K}\lambda_u^{(k)}(\text{tr}(\tilde{\mathbf{T}}_u^{(k)H}\tilde{\mathbf{T}}_u^{(k)}) - P_{\text{mask}}(f_k)) - \lambda_u^{(K+1)}(\sum_{k\in\Psi_K}\text{tr}(\tilde{\mathbf{T}}_u^{(k)H}\tilde{\mathbf{T}}_u^{(k)}) - P_{\max}) \tag{26}$$

$$L_u(\tilde{\mathbf{T}}_u,\boldsymbol{\lambda}_u) = \sum_{k\in\Psi_K}\sum_{s=1}^{M}\left(-P_{s,k}^{(u)}\text{diag}_s(\boldsymbol{\Lambda}_u^{(k)}) + \lambda_u^{(0)}\log(1+P_{s,k}^{(u)}\text{diag}_s(\boldsymbol{\Upsilon}_u^{k}))\right) + \sum_{k\in\Psi_K}\left(\frac{\lambda_u^{(K+1)}}{K}P_{\max} + \lambda_u^{(k)}P_{\text{mask}}(f_k) - \frac{\lambda_u^{(0)}}{K}c_u\right) \tag{27}$$

$$D(\boldsymbol{\lambda}_u) = \sum_{k\in\Psi_K}\left(\sum_{s=1}^{M}(\lambda_u^{(0)}\log\frac{\lambda_u^{(0)}\text{diag}_s(\boldsymbol{\Upsilon}_u^{(k)})}{\text{diag}_s(\boldsymbol{\Lambda}_u^{(k)})} - \lambda_u^{(0)} + \frac{\text{diag}_s(\boldsymbol{\Lambda}_u^{(k)})}{\text{diag}_s(\boldsymbol{\Upsilon}_u^{(k)})}) + (\frac{\lambda_u^{(K+1)}}{K}P_{\max} + \lambda_u^{(k)}P_{\text{mask}}(f_k) - \frac{\lambda_u^{(0)}}{K}c_u)\right) \tag{28}$$

$\forall s,k$ such that $\lambda_u^{(0)}\text{diag}_s(\boldsymbol{\Upsilon}_u^{(k)}) > \text{diag}_s(\boldsymbol{\Lambda}_u^{(k)}) > 0$.

$$\text{vec}(\tilde{\mathbf{T}}_u) \stackrel{\text{def}}{=} \left[\Re[vec(\tilde{\mathbf{T}}_u^{(1)})]^T, ..., \Re[vec(\tilde{\mathbf{T}}_u^{(K)})]^T, \Im[vec(\tilde{\mathbf{T}}_u^{(1)})]^T, ..., \Im[vec(\tilde{\mathbf{T}}_i^{(K)})]^T\right] \leftrightarrow \boldsymbol{x} \in \mathbb{R}^{2KMM} \tag{29}$$

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(23)) and $D(\boldsymbol{\lambda}_u)$ is the dual function, defined as:

$$D(\boldsymbol{\lambda}_u) \stackrel{\text{def}}{=} \max_{\{\tilde{\mathbf{T}}_u^{(k)},\forall k\in\Psi_K\}} L_u(\tilde{\mathbf{T}}_u,\boldsymbol{\lambda}_u). \tag{31}$$

with $L_u(\tilde{\mathbf{T}}_u,\boldsymbol{\lambda}_u)$ being the Lagrangian function (equation (26)) of problem (23).

Theorem 7: If the game (7) admits at least one NE, then at the achieved NE, the $M \times KM$ block matrix $\tilde{\mathbf{T}}_u$ that solves the individual utility optimization problem (7) (for the user's best response) must have its kth block, the matrix $\tilde{\mathbf{T}}_u^{(k)}$, in a form of the generalized eigen matrix of the matrices $\mathbf{H}_{d(u),u}^{(k)H}\mathbf{C}_{d(u)}^{(k)\,-1}\mathbf{H}_{d(u),u}^{(k)}$ and $\mathbf{I}(1+\lambda_u^{(k)}+\lambda_u^{(K+1)})+\mathbf{A}_u^{(k)}$, where $\lambda_u^{(k)}$ and $\lambda_u^{(K+1)}$ are the optimal Lagrange multipliers of (23). In other words, the following equation must hold $\forall k \in \Psi_K$ for a $M \times M$ diagonal matrix $\boldsymbol{\Pi}_i^{(k)}$:

$$\mathbf{H}_{u,u}^{(k)H}\mathbf{C}_u^{(k)\,-1}\mathbf{H}_{u,u}^{(k)}\tilde{\mathbf{T}}_u^{(k)} = [\mathbf{I}(1+\lambda_u^{(k)}+\lambda_u^{(K+1)})+\mathbf{A}_u^{(k)}]\tilde{\mathbf{T}}_u^{(k)}\boldsymbol{\Pi}_u^{(k)}. \tag{32}$$

Proof: See [27]. □

Theorem 7 provides a class of matrices that the solutions of (23) must belong to. This class tells the directions that user u should point its beams to. Specifically, from (9), if a unit-norm column matrix $\mathbf{T}_u^{(k)}$ satisfies (32), so does matrix $\tilde{\mathbf{T}}_u^{(k)}$, for variable power allocation matrices $P_{s,k}^{(u)}$. The matrix $\mathbf{T}_u^{(k)}$ can be found by normalizing the generalized eigen matrix $\tilde{\mathbf{T}}_u^{(k)}$. The next step is to find the optimal power allocation $P_{s,k}^{(u)}$ for the set of KM data streams. Since $\tilde{\mathbf{T}}_u^{(k)}$ is the generalized eigen matrix of the matrices $\mathbf{H}_{d(u),u}^{(k)H}\mathbf{C}_{d(u)}^{(k)\,-1}\mathbf{H}_{d(u),u}^{(k)}$ and $\mathbf{I}(1+\lambda_u^{(k)}+\lambda_u^{(K+1)})+\mathbf{A}_u^{(k)}$, it also diagonalizes each of the two matrices as follows [36]:

$$\begin{aligned} &\mathbf{T}_u^{(k)H}[\mathbf{H}_{u,u}^{(k)H}\mathbf{C}_u^{(k)\,-1}\mathbf{H}_{u,u}^{(k)}]\mathbf{T}_u^{(k)} = \boldsymbol{\Upsilon}_u^{(k)} \\ \text{and } &\mathbf{T}_u^{(k)H}[\mathbf{A}_u^{(k)} + (1+\lambda_u^{(K+1)}+\lambda_u^{(k)})\mathbf{I}]\mathbf{T}_u^{(k)} = \boldsymbol{\Lambda}_u^{(k)} \end{aligned} \tag{33}$$

where $\boldsymbol{\Upsilon}_u^{(k)}$ and $\boldsymbol{\Lambda}_u^{(k)}$ are $M \times M$ diagonal matrices.

Though its columns have unit-norm, $\mathbf{T}_u^{(k)}$ in general is not an orthonormal matrix, as $\mathbf{A}_u^{(k)}$ is not similar to $\mathbf{I}$. $\mathbf{T}_u^{(k)}$ (thus $\tilde{\mathbf{T}}_u^{(k)}$) does not necessarily diagonalize $\mathbf{A}_u^{(k)}$. This observation shows that though the optimal power and spectrum allocation over KM data streams seems very similar to a general water-filling problem [37] with multiple water levels (one water level per channel), it cannot be solved by the algorithms developed in [37] [38]. The general water-filling algorithm works only if $\mathbf{A}_u^{(k)}$ is a null matrix, which is game (7) without pricing.

Plugging (33) into the Lagrangian function (26), we have (27). The optimal power allocation $P_{s,k}^{(u)}$ for stream (s,k) that maximizes the above (concave) function $L_u(\tilde{\mathbf{T}}_u,\boldsymbol{\lambda}_u)$ is found at a (unique) stationary point, specified by the following equation:

$$\frac{\partial L_u(\tilde{\mathbf{T}}_u,\boldsymbol{\lambda}_u)}{\partial P_{s,k}^{(u)}} = -\text{diag}_s(\boldsymbol{\Lambda}_u^{(k)}) + \lambda_u^{(0)}\frac{\text{diag}_s(\boldsymbol{\Upsilon}_u^{k})}{1+P_{s,k}^{(u)}\text{diag}_s(\boldsymbol{\Upsilon}_u^{k})} = 0. \tag{34}$$

Thus,

$$P_{s,k}^{(u)} = \max\left(0, \frac{\lambda_u^{(0)}\text{diag}_s(\boldsymbol{\Upsilon}_u^{(k)}) - \text{diag}_s(\boldsymbol{\Lambda}_u^{(k)})}{\text{diag}_s(\boldsymbol{\Upsilon}_u^{(k)})\text{diag}_s(\boldsymbol{\Lambda}_u^{(k)})}\right). \tag{35}$$

So far, we have obtained the optimal radiation directions, the matrix $\mathbf{T}_u^{(k)}$, and the optimal power allocation $P_{s,k}^{(u)}$ as a function of a given set of Lagrangian multipliers $\lambda_u^{(k)}$. Plugging them into (27), we obtain the dual function in (28). Notice that the dual problem (with its objective function in (28)) is convex. Hence, it can be solved by standard methods (e.g., interior point or gradient/sub-gradient algorithms) for the optimal Lagrangian multipliers $\lambda_u^{(k)}$. The above analysis is summarized in Algorithm 1. We emphasize that by exploiting the strong duality, this algorithm needs only to deal with $K+1$ variables, instead of $2KM^2$ variables for the primal problem (23).

Another issue is whether the game converges to a NE while running Algorithm 1. The convergence speed depends on how players update their response. Typically, there are two types of updating mechanisms: synchronous (sequential/Gauss-Seidel and parallel/Jacobi) and asynchronous updates. In the sequential update, players take turns in updating their parameters. In the parallel update, all players respond simultaneously (see Algorithm 1). To maintain synchronous updates, nodes have to be in synch and honor the updating rule. This can be realized by any coarse synchronization method. In contrast, in

Algorithm 1 Distributed algorithm to compute the best precoders $\tilde{\mathbf{T}}_u(t+1)$ at node u and time $(t+1)$

Input:
$\tilde{\mathbf{T}}_{-u} = [\tilde{\mathbf{T}}_1(t+1), ..., \tilde{\mathbf{T}}_{u-1}(t+1), \tilde{\mathbf{T}}_{u+1}(t), ..., \tilde{\mathbf{T}}_N(t)]$ with Gauss-Seidel iteration
$\tilde{\mathbf{T}}_{-u} = [\tilde{\mathbf{T}}_1(t), ..., \tilde{\mathbf{T}}_{u-1}(t), \tilde{\mathbf{T}}_{u+1}(t), ..., \tilde{\mathbf{T}}_N(t)]$ with Jacobi iteration
Initialize
$\tilde{\mathbf{T}}_u^{(k)}(t+1) \leftarrow \tilde{\mathbf{T}}_{\mathbf{u}}^{(\mathbf{k})}(\mathbf{t}), \boldsymbol{\lambda}_u \leftarrow \mathbf{0}$
while $true$ **do**
    Iteratively solve the dual problem (30)
    If the duality gap is zero, **break**
end while
Plug $\boldsymbol{\lambda}_u$ into (32) (Theorem 7) to find $\mathbf{T}_u^{(k)}$ by nomarlizing the generalized eigen matrix. $\mathbf{P}_{s,k}^{(i)}$ is found from (33), (35). Optimal precoders $\tilde{\mathbf{T}}_u^{(k)}$ is found from (9).
RETURN $\tilde{\mathbf{T}}_u^{(k)}(t+1), \forall k \in \Psi_K$ at time $(t+1)$

an asynchronous update, nodes send their responses following an arbitrary order or even skip response for a finite duration. Though we cannot prove the convergence under the Jacobi and asynchronous iterations, simulations show that Algorithm 1 converges under all above updating policies. The convergence under the Gauss-Seidel iteration is claimed in the following theorem.

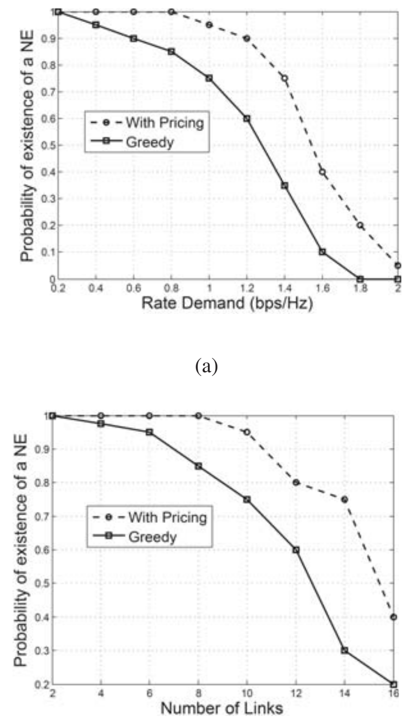
Theorem 8: Under the sequential update (Gauss-Seidel), Algorithm 1 drives the game (23) to its unique NE.

Proof: See [27]. □

VI. Numerical Results

In this section, we numerically evaluate the conditions for the existence and uniqueness of a NE of the distributed algorithm in Theorem 2 and the effectiveness of the proposed joint beamforming and power/spectrum management. Nodes are equipped with 4 antennas. The simulation results are averaged over 40 runs. In each run, N links are randomly placed in a square area of length 100 meters. We set $P_{\max} = 1000$ mW and the power mask $P_{\text{mask}} = 0.5P_{\max}$ for all channels. The channel fading is flat with free-space attenuation factor of 2. The spreading angles of the signal at the receive antennas vary from $-\pi/5$ to $\pi/5$. The channel bandwidth is 16 MHz. The close-in distance is 1 m. The thermal floor noise is -174 dBm/Hz. The PUs interference on all channels is -100 dBm/Hz. We also assume that links have identical rate demands.

For a given simulation run, there is a probability that the conditions in Theorem 2 hold and the game converges to a unique NE. Recall that these conditions are sufficient but not necessary, so when these conditions do not hold, a unique NE may still exist. Fig. 1(a) depicts the probability (percentage of runs) that the game converges to a NE (a NE exists) versus the rate profile when 10 links are active and 10 channels are used. As the rate demand increases, the probability that a NE exists decreases. This is because the conditions in Theorem 2 become more stringent. Fig. 1(b) depicts the probability that a NE exists versus N when the rate demand is 1 bps/Hz. As N increases, the network/multi-user interference becomes more severe, so there is a less chance of meeting the conditions in Theorem 2. Thus, the probability of a NE existence decreases.



(a)

(b)

Fig. 1. (a) Probability of NE existence vs. rate demands, (b) Probability of NE existence vs. number of links.

In both Fig. 1(a) and 1(b), the distributed algorithm with pricing has a higher chance of converging to a NE than the one without pricing. This sounds counter-intuitive, as both games have the same sufficient conditions for the existence of a NE. However, the proposed pricing function helps reduce the required power (see below). Moreover, the conditions in Theorem 2 are sufficient but not necessary. Then, when these conditions do not hold, there is still a higher chance for the lower-power consumption algorithm to secure a NE than the one that requires higher power.

To evaluate the total power consumption, we simulate a network of 8 links with a rate demand of 1 bps/Hz. Other parameters are as above. Fig. 2 compares the total required power (averaged over converged runs) under the game (7) and the game (23) (with pricing) with two other algorithms. The first one is when we evenly divide the total power budget and the rate demand over all available bands and separately apply the pricing policy with the pricing factor matrix $\mathbf{A}_u^{(k)}$ to each band f_k, referred to as "Sept Opt With Pricing". The second one is obtained by dividing the power and rate demand evenly over all channels then applying the approach derived under full-duplex assumption in [16] while setting the rate demand on one direction to zero, referred to as "Sept Opt FD". During the simulations, we observed that the probability that algorithm "Sept Opt FD" does not converge is significantly higher than that of the three others, suggesting its instability. For converged cases, "Sept Opt FD" requires the most power. This is because, "Sept Opt FD" was developed with an implicit assumption of full-duplex MIMO transceivers. Using the proposed pricing policy ("Joint Opt With Pricing"), the total network power can be reduced by 25%, compared with algorithm "Joint Opt

Without Pricing" (pricing not used). Comparing the required power under "Joint Opt With Pricing" and "Sept Opt With Pricing" shows the superior power efficiency of joint optimizing power and spectrum allocation. Both games (with and without pricing) converge to their NEs after about 8 iterations.

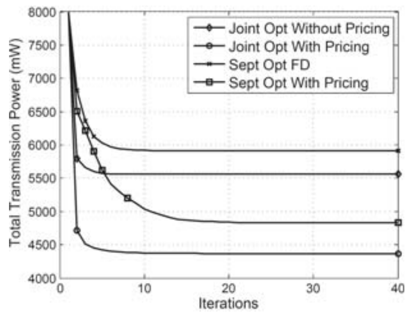


Fig. 2. Total network power consumption vs. iterations.

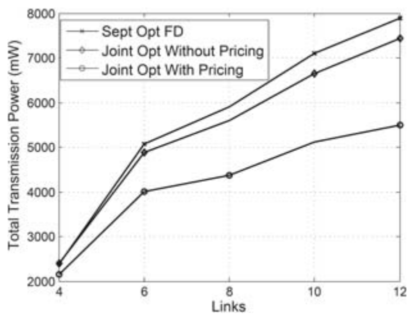


Fig. 4. Total network power consumption vs. number of links.

Fig. 4 compares the required power under game (7) (without pricing) and game (23) (with pricing) vs. N. When N increases, pricing helps to conserve more power. This is because as interference becomes more severe, the pricing policy is more helpful to control transmitters' radiation beams. This fact is further demonstrated in Fig. 3, which depicts two snapshots of the network topology and radiation patterns (on a representative channel) with and without pricing. Visually, compared with the case when pricing is not used, transmitters using the proposed pricing policy cause less interference to their unintended receivers by steering their beams away from these receivers (highlighted in ovals).

Fig. 5 shows the averaged number of iterations before reaching the NE under both synchronous (Jacobi and Gauss-Seidel) and asynchronous updating methods. For asynchronous updating, we allow odd-numbered links skip their updates every other iteration and even-numbered links skip their updates once every 3 iterations. As we can see, the game still converges to the NE under asynchronous update although its speed is slower than synchronous updates. When all players update their strategies simultaneously (Jacobi), the game converges faster. The difference in convergence speed of the Jacobi and Gauss-Seidel updates becomes more significant with the increase in the number of players.

VII. Conclusions

In this work, we aimed at improving the energy and spectrum efficiency of MIMO dynamic spectrum networks. This

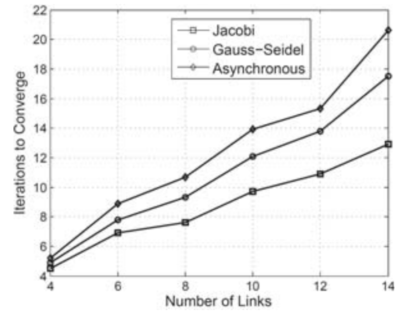


Fig. 5. Convergence speed vs. number of CR links.

was done by jointly optimizing the beamformers, power, and spectrum allocation for each link to minimize the total transmit power subject to rate demands. Using game theory, variational inequalities theory, and recession analysis, we derived sufficient conditions for the existence and uniqueness of the NE of the game. By exploiting the strong duality in convex optimization, we designed a low-complexity distributed algorithm that allows nodes to optimally determine their radiation patterns and power allocation. We then proposed pricing policies that use a novel network interference function. Using this pricing policy, the game also converges to a unique NE with significantly improved efficiency.

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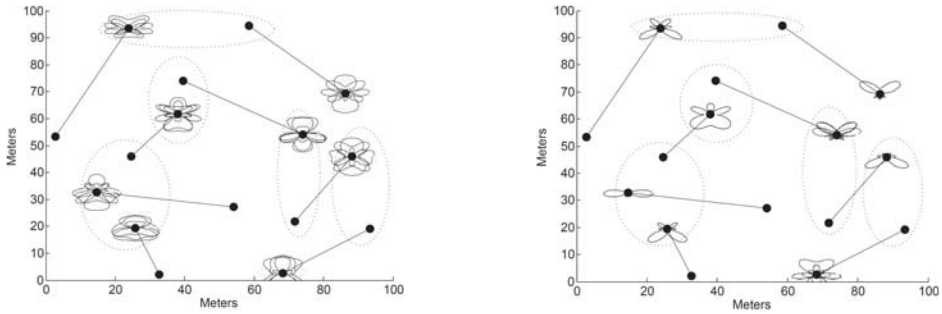

(a) (b)

Fig. 3. (a) Radiation patterns when not using pricing and (b) when using pricing. Pricing is effective in steering radiation beams from unintended receivers.

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Diep N. Nguyen received the B.S. degree in telecommunications and electronics from Posts and Telecommunications Institute of Technology (PTIT), Vietnam, in 2004, and the M.E. degree in electrical and computer engineering from the University of California, San Diego in 2008. From 2004 to 2006, he was a lecturer at the Department of Telecommunications, PTIT, Vietnam. He was a member of technical staff at Broadcom Corporation from 2008 to 2009. He is currently working towards the Ph.D. degree in Electrical and Computer Engineering at The University of Arizona. His research interests include cognitive radios, MIMO, sensor networks, game theory, and network optimization.

Marwan Krunz is a professor of ECE and CS at the University of Arizona. He received his Ph.D. degree in electrical engineering from Michigan State University in 1995. He previously held visiting research positions at INRIA, HP Labs, University of Paris VI, and US West Advanced Technologies. In 2010, he was a visiting researcher at Institute IMDEA and a Chair of Excellence at the University of Carlos III, Madrid, Spain. Dr. Krunz’s research interests lie in the fields of computer networking and wireless communications, with focus on distributed radio resource management in wireless and sensor networks; protocol design; and secure communications. He has published more than 190 journal articles and refereed conference papers, and is a co-inventor on three US patents. He is a recipient of the IEEE 2012 Communications Societys TCCC Outstanding Service Award and the NSF CAREER Award (1998). He currently serves on the editorial board for the IEEE Transactions on Network and Service Management. Previously, he served on the editorial boards for the IEEE/ACM Transactions on Networking, the IEEE Transactions on Mobile Computing, and the Computer Communications Journal. He was the general co-chair for the ACM WiSec 2012 Conference, and served as a TPC chair for various international conferences, including INFOCOM04, SECON05, and WoWMoM06. He is an IEEE Fellow and an Arizona Engineering Faculty Fellow.