Distributed Bargaining Mechanisms for Multi-antenna Dynamic Spectrum Access Systems

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***Abstract*—Dynamic spectrum access and MIMO technologies are among the most promising solutions to address the ever increasing wireless traffic demand. An integration that successfully embraces the two is far from trivial due to the dynamics of spectrum opportunities as well as the requirement to jointly optimize both spectrum and spatial/antenna dimensions. Our objective in this paper is to jointly allocate opportunistic channels to various links such that no channel is allocated to more than one link, and to simultaneously optimize the MIMO precoding matrices under the Nash bargaining (NB) framework. We design a low-complexity distributed scheme that allows links to propose their minimum rate requirements, negotiate the channel allocation, and configure their precoding matrices. Simulations confirm the convergence of the distributed algorithm under timesharing to the globally optimal solution of the NB-based problem. They also show that the NB-based algorithm achieves much better fairness than purely maximizing network throughput.**

***Index Terms*—Nash bargaining, dual decomposition, distributed algorithm, cognitive radio, MIMO precoding, fairness, rate demands.**

I. INTRODUCTION

Dynamic spectrum access (DSA) and multi-input multi-output (MIMO) communications have been at the forefront of communications research. Newly emerging systems and standards (e.g., 4G Advanced-LTE, IEEE 802.16e, IEEE 802.11ac) adopt MIMO as a core technology. The FCC has opened up TV white bands for opportunistic use [1]. A timely issue is how to embrace recent innovations of the two technologies into a single system.

In this work, we design both centralized and distributed algorithms that allow MIMO-capable secondary users, referred to as cognitive MIMO (CMIMO) nodes, to cooperate/bargain for the purpose of determining their assigned channels and optimally designing their Tx/Rx beamformers under the a *heterogeneous* spectrum scenario (i.e., the set of available channels varies from one link to another). We follow a Nash bargaining (NB) approach [2] and propose an NB scheme for CMIMO systems, referred to as BF-CMIMO (bargaining framework for CMIMO). NB-based resource allocation often yields superior performance than noncooperative ones [3] [4] [5].

Existing NB solutions (e.g., [3] [4] [5] [6] [7] [8]) are often centralized and require an *arbitrator* to manage the bargaining process. The only fully distributed NB design was provided in [9], but under the assumption of an unlimited number of available channels that is unrealistic in DSA systems. Moreover, almost all of the NB schemes in the literature were developed for single-antenna systems, with the exception of [8] which was developed for MIMO downlink communications. However, the algorithm in [8] is centralized and does not support an exclusive channel occupancy policy (i.e., a channel is assigned to no more than one interfering link). The challenge that hinders a fully distributed algorithm is the combinatorial complexity of the joint power/channel allocation problem, which includes integer and real variables. Relaxing the integer variables does not make the problem convex.

To overcome the aforementioned challenges, we start with a BF-CMIMO formulation and transform it to an equivalent one whose relaxed version is convex. The relaxed version of the transformed problem is two-fold. First, the relaxed variable can be interpreted as a "timesharing factor" that represents the fraction of time a channel is allocated to a link. Hence, this relaxed version is of practical interest when time-synchronization among links is possible. An arbitrator-assisted (centralized) bargaining algorithm is then developed for the timesharing scenario. Using dual decomposition [10], a distributed algorithm for the timesharing problem is developed and proved to drive the bargaining process to the globally optimal solution. Second, the distributed bargaining algorithm under timesharing gauges the preferences of different CMIMO links of on a channel (quantified by a "payoff" vector). Using these preferences, a distributed heuristic algorithm for the original BF-CMIMO is derived.

Throughout the paper, we use $(.)^H$ for the Hermitian matrix transpose, tr$(.)$ for the trace of a matrix, $|.|$ for the determinant, and $\text{eig}_{\max}(.)$ for the maximum eigenvalue of a matrix. Matrices and vectors are indicated in boldface.

II. PROBLEM SETUP

A. Network Model

Consider a CMIMO network of N links with M antennas per node. The set of currently idle channels for link i is denoted by $\mathbf{S}_i$. In general, $\mathbf{S}_i \neq \mathbf{S}_j$ for two links i and j, although due to their proximity the two links are likely to share several idle channels. Without loss of generality, we assume that $\Psi_K \stackrel{\text{def}}{=} \{1, 2, \dots, K\} = \bigcup_{i=1}^{N} \mathbf{S}_i$ consists of K orthogonal (not necessarily contiguous) channels with central frequencies f_1, f_2, $\dots$, f_K (for simplicity, we use the same notation f_k to refer to the kth channel). Let $\Phi_N \stackrel{\text{def}}{=} \{1, 2, \dots, N\}$ denote the sets of links and channels. At a given time instant, each link i may simultaneously communicate over a subset of channels in $\mathbf{S}_i$, denoted by $\mathbf{A}_i$. However, a channel cannot be allocated to more than one link, i.e., $\mathbf{A}_i \cap \mathbf{A}_j = \emptyset, \forall i \neq j$. This requirement is called *exclusive channel occupancy*, which goes in line with the so-called "protocol model". Let $\mathbf{A} = [a_{i,k}]$ be an $N \times K$ where $a_{i,k} = 1$ if channel f_k is allocated to link i, otherwise $a_{i,k} = 0$. On channel f_k, let $\mathbf{x}_{i,k}$ be a column vector of M information symbols, sent from transmitter i to its receiver. Each element of $\mathbf{x}_{i,k}$ is from one data stream. Let $\tilde{\mathbf{T}}_{i,k} \in \mathbb{C}^{M \times M}$ denote the

This research was supported in part by NSF (under grants CNS-0904681, IIP-0832238, and IIP-1231043), Raytheon, and the Connection One center. Any opinions, findings in this paper are those of the author(s) and do not necessarily reflect the NSF views.

precoding matrix of transmitter i on channel f_k. Then, the actual transmit vector is $\tilde{\mathbf{T}}_{i,k}\mathbf{x}_{i,k}$. For channel f_k, the received signal vector $\mathbf{y}_{i,k}$ at the receiver of link i is given by:

$$\mathbf{y}_{i,k} = \mathbf{H}_{i,i}^{(k)}\tilde{\mathbf{T}}_{i,k}\mathbf{x}_{i,k} + \mathbf{N}_k \tag{1}$$

where $\mathbf{H}_{i,i}^{(k)}$ is an $M \times M$ channel gain matrix for channel f_k on link i and $\mathbf{N}_k \in \mathbb{C}^M$ is an $M \times 1$ complex Gaussian noise vector with identity covariance matrix $\mathbf{I}$, representing the floor noise plus normalized (and whitened) interference from PUs on channel k. Each element of $\mathbf{H}_{i,i}^{(k)}$ is the multiplication of a distance- and channel-dependent attenuation term, and a random term that reflects multi-path fading (a complex Gaussian variable with zero mean and unit variance). We assume a flat-fading channel. The Shannon rate for link i on channel f_k is [11]:

$$R_{i,k} = \log|\mathbf{I} + \tilde{\mathbf{T}}_{i,k}^H \mathbf{H}_{i,i}^{(k)H}\mathbf{H}_{i,i}^{(k)}\tilde{\mathbf{T}}_{i,k}|. \tag{2}$$

The total channel rate over all channels assigned to link i is $R_i = \sum_{k\in\mathbf{S}_i} a_{i,k}R_{i,k}$. Each link i is subject to a rate demand c_i, i.e., we require that $R_i \geq c_i$. Let $P_{s,k}^{(i)}$ denote the allocated power on channel k and antenna s of link i. For link i, the total power allocated on all channels and all antennas should not exceed a maximum power budget $P_{\max}$:

$$\sum_{k\in\mathbf{S}_i}\sum_{s=1}^{M} P_{s,k}^{(i)} = \sum_{k\in\mathbf{S}_i} \text{tr}(\tilde{\mathbf{T}}_{i,k}^H\tilde{\mathbf{T}}_{i,k}) \leq P_{\max}. \tag{3}$$

PU protection is provided in the form of database-authorized access and frequency-dependent power masks on secondary transmissions. In its recent specifications [1], the FCC has imposed power masks on opportunistic transmissions even over idle channels, if such channels are adjacent to PU-active channels. Let $\mathbf{P}_{\text{mask}}^* \stackrel{\text{def}}{=} (P_{\text{mask}}^*(f_1), P_{\text{mask}}^*(f_2), \ldots, P_{\text{mask}}^*(f_K))$ denote the vector of power masks. We require:

$$\sum_{s=1}^{M} P_{s,k}^{(i)} = \text{tr}(\tilde{\mathbf{T}}_{i,k}^H\tilde{\mathbf{T}}_{i,k}) \leq P_{\text{mask}}^*(f_k), \forall i \text{ and } \forall k. \tag{4}$$

To accommodate spectrum heterogeneity, we force link i not to transmit on channels that are not available for its use by imposing a link-dependent power-mask vector as in [12] $\mathbf{P}_{\text{mask}}^*(i)$. For link i, $\mathbf{P}_{\text{mask}}^*(i) \stackrel{\text{def}}{=} (P_{\text{mask}}^*(i,f_1), P_{\text{mask}}^*(i,f_2), \ldots, P_{\text{mask}}^*(i,f_K))$, where $P_{\text{mask}}^*(i,f_k) = 0$ if $f_k \notin \mathbf{S}_i$, and $P_{\text{mask}}^*(i,f_k) = P_{\text{mask}}^*(f_k)$ otherwise. Note that $\mathbf{P}_{\text{mask}}^*(i)$ differs from one link to another.

B. Problem Formulation

We propose a Nash bargaining framework for CMIMO networks, called BF-CMIMO. In this framework, nodes first announce their rate demands and then jointly select their channels and optimize their precoders in a distributed manner. Nash [2] proposed axioms that define a Nash bargaining solution (NBS). An NBS guarantees all users' demands and is Pareto optimal, meaning that there is no other solution that leads to better payoffs for two or more players simultaneously.

Theorem 1: [2] If the utility space U is upper-bounded, closed, and convex, then there exists a unique NBS, which is obtained by solving the following problem:

$$\max_{\{\boldsymbol{b}\in B\}} \prod_{i=1}^{N}(u_i - u_i^0). \tag{5}$$

where b and B are action set and action space of all players. u_i and u_i^0 are the achieved utility (e.g., throughput) and the utility demand of player/link i. The utility space U is the set of all possible payoff allocations. Even if U is not convex, the NBS may still exist. Though a convex utility space makes the bargaining process more tractable, cases with nonconvex utility spaces (e.g., the one in this paper) are common.

In the CMIMO setup, each transmitting node is a player. The action of player i is $(\mathbf{A}_i, \tilde{\mathbf{T}}_i)$ where $\tilde{\mathbf{T}}_i \stackrel{\text{def}}{=} \{\tilde{\mathbf{T}}_{i,k}, k \in \mathbf{A}_i\}$ is the set of precoding matrices for the set of channels allocated to i. We aim at finding a channel allocation matrix $\mathbf{A}$ and sets of precoders for all CR transmitters ($\tilde{\mathbf{T}}_i, \forall i \in \Phi_N$) that solve the following problem:

$$\begin{aligned}
&\underset{\{a_{i,k},\tilde{\mathbf{T}}_{i,k},\forall k\in\mathbf{S}_i,\forall i\in\Phi_N\}}{\text{maximize}} \sum_{i\in\Phi_N}\log(\sum_{k\in\mathbf{S}_i} a_{i,k}R_{i,k} - c_i)\\
&\text{s.t.}\quad \text{C1: } \sum_{k\in\Psi_K}\text{tr}(\tilde{\mathbf{T}}_{i,k}^H\tilde{\mathbf{T}}_{i,k}) \leq P_{\max}, \forall i\in\Phi_N\\
&\qquad\ \text{C2:}\text{tr}(\tilde{\mathbf{T}}_{i,k}^H\tilde{\mathbf{T}}_{i,k}) \leq P_{\text{mask}}^*(i,f_k), \forall k\in\Psi_K, \forall i\in\Phi_N\\
&\qquad\ \text{C3: } \sum_{k\in\Psi_K} a_{i,k}R_{i,k} \geq c_i, \forall i\in\Phi_N\\
&\qquad\ \text{C4: } \sum_{i\in\Phi_N} a_{i,k} \leq 1, \forall k\in\Psi_K\\
&\qquad\ \text{C5:} a_{i,k} = \{0,1\}, \forall k\in\Psi_K, \forall i\in\Phi_N
\end{aligned} \tag{6}$$

where the objective function is mapped to that of the NBS in (5).

Because each channel can be assigned to one link only, the best strategy for the transmitter and receiver of a given MIMO link is to design their beamformers so that their M data streams do not interfere with each other [11]. These beamformers can be derived from the CSI matrix using singular-value decomposition:

$$\mathbf{H}_{i,i}^{(k)} = \mathbf{U}_{i,k}\mathbf{G}_{i,k}\mathbf{T}_{i,k}^H \tag{7}$$

where $\mathbf{U}_{i,k}$ and $\mathbf{T}_{i,k}$ are unitary matrices, and $\mathbf{G}_{i,k}$ is a diagonal matrix formed from the singular values $g_{s,k}^{(i)}$, $s = 1, \ldots, M$, of the channel gain matrix $\mathbf{H}_{i,i}^{(k)}$. At the transmitter, we set $\tilde{\mathbf{T}}_{i,k}$ to $\mathbf{T}_{i,k}\mathbf{P}_k^{(i)^{1/2}}$ [11], where $\mathbf{P}_k^{(i)}$ is a diagonal matrix whose sth diagonal element is $P_{s,k}^{(i)}$. The achievable rate over channel f_k is $R_{i,k} = \sum_{s=1}^{M}\log(1 + g_{s,k}^{(i)}P_{s,k}^{(i)})$. We can rewrite (6) as follows:

$$\begin{aligned}
&\underset{\{a_{i,k},P_{s,k}^{(i)}\}}{\text{maximize}} \sum_{i\in\Phi_N}\log(\sum_{k\in\Psi_K} a_{i,k}\sum_{s=1}^{M}\log(1+g_{s,k}^{(i)}P_{s,k}^{(i)}) - c_i)\\
&\text{s.t.}\quad \text{C1': } \sum_{k\in\Psi_K}\sum_{s=1}^{M} P_{s,k}^{(i)} \leq P_{\max}, \forall i\in\Phi_N\\
&\qquad\ \text{C2': } \sum_{s=1}^{M} P_{s,k}^{(i)} \leq P_{\text{mask}}^*(i,f_k), \forall k\in\Psi_K, \forall i\in\Phi_N\\
&\qquad\ \text{C3': } \sum_{k\in\Psi_K} a_{i,k}\sum_{s=1}^{M}\log(1 + g_{s,k}^{(i)}P_{s,k}^{(i)}) \geq c_i, \forall i\in\Phi_N\\
&\qquad\ \text{C4': } \sum_{i\in\Phi_N} a_{i,k} \leq 1, \forall k\in\Psi_K\\
&\qquad\ \text{C5':} a_{i,k} = \{0,1\}, \forall k\in\Psi_K, \forall i\in\Phi_N.
\end{aligned} \tag{8}$$

III. Distributed Bargaining Algorithm

A. Convexification and Timesharing Interpretation

Problem (8) is NP-hard [13]. If we relax the binary constraint C5', its relaxed version is not convex as the objective function is not concave w.r.t. $(a_{i,k}, P_{s,k}^{(i)})$. To address (8) and provide a distributed algorithm, lets consider the following function:

$$f(a_{i,k},P_{s,k}^{(i)})\stackrel{\text{def}}{=}\begin{cases}a_{i,k}\sum\limits_{s=1}^{M}\log(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}}) & \text{if } 0<a_{i,k}\le 1\\ 0 & \text{if } a_{i,k}=0.\end{cases} \quad (9)$$

It is easy to verify that the bargaining problem (8) is equivalent to:

$$\begin{aligned}&\underset{\{a_{i,k},P_{s,k}^{(i)}\}}{\text{maximize}} \sum_{i\in\Phi_N}\log(\sum_{k\in\mathbf{S}_i} f(a_{i,k},P_{s,k}^{(i)})-c_i)\\ &\text{s.t.} \quad \text{C1', C2', C3', C4', C5' in (8).}\end{aligned} \quad (10)$$

A relaxed version of (10) can be written as:

$$\begin{aligned}&\underset{\{a_{i,k},P_{s,k}^{(i)}\}}{\text{maximize}} \sum_{i\in\Phi_N}\log(\sum_{k\in\Psi_K} f(a_{i,k},P_{s,k}^{(i)})-c_i)\\ &\text{s.t.} \quad \text{C1', C2', C3', C4' in (8)}\\ &\qquad 0\le a_{i,k}\le 1,\ \ \forall k\in\Psi_K,\forall i\in\Phi_N.\end{aligned} \quad (11)$$

The advantage of (10) over (8) is that its relaxed version (11) is convex w.r.t. $(a_{i,k},P_{s,k}^{(i)})$.

Theorem 2: Problem (11) is a convex optimization problem.

Proof: See Appendix A in [14]. □

Problem (11) itself is practically useful if transmissions are time-synchronized. The relaxed variable $a_{i,k}$ can be interpreted as the fraction of time that link i is allowed to use channel f_k [15]. Under the timesharing assumption, the convex problem (11) complies with Theorem 1, hence a unique and Pareto-optimal NBS is the solution of (11).

Theorem 3: If timesharing is allowed, then a unique NBS exists and is the solution to problem (11).

B. Distributed Optimal Algorithm using Dual Decomposition

The bargaining formulation (11) under timesharing is convex and its Slater's conditions hold [16]. Hence, strong duality holds, meaning that the solution of its dual problem also solves the primal problem (11). The Lagrangian of (11) is given in (12), where $\alpha_{i,k}$, γ_i, β_i, and ρ_k are nonnegative Lagrangian multipliers, interpreted as *prices* for violating the constraints. The dual problem of (11) is:

$$\text{DP}: \underset{\{\alpha_{i,k},\gamma_i,\beta_i,\rho_k,\forall k\in\mathbf{S}_i,\forall i\in\Psi_N\}}{\text{minimize}} D(\alpha_{i,k},\gamma_i,\beta_i,\rho_k) \quad (16)$$

where D is the dual function, defined as:

$$D=\max_{\{a_{i,k},P_{s,k}^{(i)},\forall k\in\mathbf{S}_i,\forall i\in\Psi_N\}} L(a_{i,k},P_{s,k}^{(i)},\alpha_{i,k},\gamma_i,\beta_i,\rho_k). \quad (17)$$

To facilitate a distributed solution, we decompose the Lagrangian of the primal problem in (13) with:

$$\begin{aligned}&L_i(a_{i,k},P_{s,k}^{(i)},\alpha_{i,k},\gamma_i,\beta_i,\rho_k)\\ &=\log(\sum_{k\in\Psi_K}a_{i,k}\sum_{s=1}^{M}\log(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}})-c_i)\\ &+\sum_{k\in\Psi_K}\alpha_{i,k}(-\sum_{s=1}^{M}P_{s,k}^{(i)}+P^{*}_{\text{mask}}(i,f_k))+\gamma_i(-\sum_{k\in\Psi_K}\sum_{s=1}^{M}P_{s,k}^{(i)}+P_{\max})\\ &+\beta_i(\sum_{k\in\Psi_K}a_{i,k}\sum_{s=1}^{M}\log(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}})-c_i)-\sum_{k\in\Psi_K}\rho_k a_{i,k}.\end{aligned} \quad (18)$$

To solve (17) for the dual function, each link individually maximizes $L_i(a_{i,k},P_{s,k}^{(i)},\alpha_{i,k},\gamma_i,\beta_i,\rho_k)$ to find the optimal $(a_{i,k}^{*},P_{s,k}^{(i)*})$ for given prices $(\alpha_{i,k},\gamma_i,\beta_i,\rho_k)$:

$$\underset{\{a_{i,k}\ge 0,P_{s,k}^{(i)}\ge 0,\forall k\in\Psi_K\}}{\text{maximize}} L_i(a_{i,k},P_{s,k}^{(i)},\alpha_{i,k},\gamma_i,\beta_i,\rho_k). \quad (19)$$

The local problem (19) is convex, and hence can be solved using standard methods like "interior fixed point". If a central arbitrator is in place (e.g., a base station or spectrum database/broker), after solving the local problem (19), all links report their calculated $(a_{i,k}^{*},P_{s,k}^{(i)*})$ to the arbitrator so that the dual function is updated as $L(a_{i,k}^{*},P_{s,k}^{(i)*},\alpha_{i,k},\gamma_i,\beta_i,\rho_k)$.

Because the dual problem DP (16) is convex [10], the arbitrator can solve it efficiently for $(\alpha_{i,k},\gamma_i,\beta_i,\rho_k)$, and then broadcasts these variables. Each link updates its local problem (19) with broadcasted Lagrangian variables. This process is illustrated in Fig. 1 and referred to as "Arbitrator-Assisted Scheme".

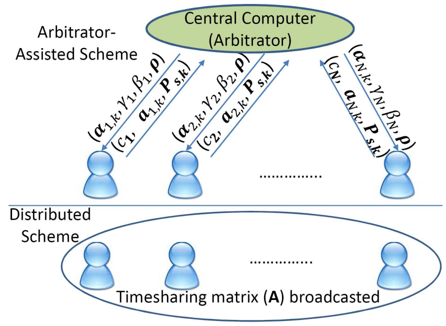

Fig. 1. Arbitrator-assisted and distributed bargaining schemes.

Next, we design a fully distributed and optimal algorithm for problem (11) (i.e., no central controler/arbitrator is available). Since the dual problem is convex and its objective function is differentiable, DP can be solved with a gradient search algorithm. The DP's variables at time $(t+1)$ are updated as follows:

$$\begin{aligned}&\alpha_{i,k}^{(t+1)}=\left[\alpha_{i,k}^{(t)}-\eta\frac{\partial L}{\partial\alpha_{i,k}}\right]^{+}=\left[\alpha_{i,k}^{(t)}-\eta(-\sum_{s=1}^{M}P_{s,k}^{(i)(t)*}+P^{*}_{\text{mask}}(i,f_k))\right]^{+}\\ &\gamma_i^{(t+1)}=\left[\gamma_i^{(t)}-\eta\frac{\partial L}{\partial\gamma_i}\right]^{+}=\left[\gamma_i^{(t)}-\eta(-\sum_{k\in\Psi_K}\sum_{s=1}^{M}P_{s,k}^{(i)(t)*}+P_{\max})\right]^{+}\\ &\beta_i^{(t+1)}=\left[\beta_i^{(t)}-\eta\frac{\partial L}{\partial\beta_i}\right]^{+}\\ &\qquad=\left[\beta_i^{(t)}-\eta(\sum_{k\in\Psi_K}a_{i,k}^{(t)*}\sum_{s=1}^{M}\log(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)(t)*}}{a_{i,k}^{(t)*}})-c_i)\right]^{+}\\ &\rho_k^{(t+1)}=\left[\rho_k^{(t)}-\eta\frac{\partial L}{\partial\rho_k}\right]^{+}=\left[\rho_k^{(t)}-\eta(-\sum_{i\in\Phi_N}a_{i,k}^{(t)*}+1)\right]^{+}\end{aligned} \quad (20)$$

where $\eta>0$ is a sufficiently small step size and $[.]^{+}$ denotes the projection onto the nonnegative orthant.

Observe that the Lagrangian variables $\alpha_{i,k}$, γ_i, and β_i can be calculated and updated using only local information of link i (the fraction of time $a_{i,k}$ that link i wishes to communicate on channel f_k and the power allocated to stream s on channel f_k, $P_{s,k}^{(i)}$). Moreover, the price ρ_k is obtained if other links j broadcast their timeshare $a_{j,k}$ on channel f_k. Our fully distributed mechanism is shown in Algorithm 1 and illustrated in Fig. 1. The key idea

$$L(a_{i,k}, P_{s,k}^{(i)}, \alpha_{i,k}, \gamma_i, \beta_i, \rho_k) = \sum_{i\in\Phi_N} \log(\sum_{k\in\mathbf{S}_i} a_{i,k} \sum_{s=1}^{M} \log(1+\frac{g_{s,k}^{(i)} P_{s,k}^{(i)}}{a_{i,k}}) - c_i) + \sum_{i\in\Phi_N}\sum_{k\in\Psi_K} \alpha_{i,k}[-\sum_{s=1}^{M} P_{s,k}^{(i)} + P_{mask}(i,f_k)]$$

$$+ \sum_{i\in\Phi_N} \gamma_i[-\sum_{k\in\Psi_K}\sum_{s=1}^{M} P_{s,k}^{(i)} + P_{\max}] + \sum_{i\in\Phi_N} \beta_i[\sum_{k\in\Psi_K} a_{i,k}\sum_{s=1}^{M}\log(1+\frac{g_{s,k}^{(i)} P_{s,k}^{(i)}}{a_{i,k}}) - c_i] + \sum_{k\in\Psi_K} \rho_k(-\sum_{i\in\Phi_N} a_{i,k} + 1) \tag{12}$$

$$= \sum_{i\in\Phi_N} L_i(a_{i,k}, P_{s,k}^{(i)}, \alpha_{i,k}, \gamma_i, \beta_i, \rho_k) + \sum_{k\in\Psi_K} \rho_k \tag{13}$$

$$\frac{\partial L}{\partial P_{s,k}^{(i)}} = \frac{g_{s,k}^{(i)}}{\left(\sum\limits_{k\in\Psi_K} a_{i,k}\sum\limits_{s=1}^{M}\log(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}}) - c_i\right)(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}})} - \alpha_{i,k} - \gamma_i + \beta_i \frac{g_{s,k}^{(i)}}{(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}})} \begin{cases} = 0 & \text{if } P_{s,k} > 0 \\ < 0 & \text{if } P_{s,k}^{(i)} = 0 \end{cases} \tag{14}$$

$$\frac{\partial L}{\partial a_{i,k}} = \frac{\sum\limits_{s=1}^{M}\left(\log(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}}) - a_{i,k}\frac{\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}^2}}{(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}})}\right)}{\left(\sum\limits_{k\in\Psi_K} a_{i,k}\sum\limits_{s=1}^{M}\log(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}}) - c_i\right)} + \beta_i\sum_{s=1}^{M}\left(\log(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}}) - a_{i,k}\frac{\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}^2}}{(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}})}\right) - \rho_k \begin{cases} = 0 & \text{if } 0 < a_{i,k} < 0 \\ > 0 & \text{if } a_{i,k} = 1 \\ < 0 & \text{if } a_{i,k} = 0 \end{cases} \tag{15}$$

in Algorithm 1, inline with the Network Utility Maximization (NUM) problem in [17], is to ignore the iterations of updates in (20), which would have been carried out by an arbitrator. However, we prove that this simplification does not affect the convergence and optimality of Algorithm 1.

Algorithm 1 Distributed Bargaining Algorithm for Computing Optimal Timeshares and Precoders of Link i at Time $(t+1)$:

Input: $\boldsymbol{a} = (a_{1,k}^{(t+1)}, ..., a_{i-1,k}^{(t+1)}, a_{i+1,k}^{(t)}, ..., a_{N,k}^{(t)}), \forall k \in \Psi_K$
If $t+1=0$ (beginning iteration), set $\boldsymbol{a} = (1/N, \ldots, 1/N)$
Initialize: $\tilde{\mathbf{T}}_i^{(t+1)} \leftarrow \tilde{\mathbf{T}}_i^{(t)}$
Computation:
$\forall k \in \Psi_K$, compute transmit and receive beamformers ($\mathbf{T}_{i,k}$, $\mathbf{U}_{i,k}^H$), and stream gains $g_{s,k}^{(i)}$ using (7).
Update local Lagrangian variables $\alpha_{i,k}^{(t+1)}$, ${\gamma_i}^{(t+1)}$, and ${\beta_i}^{(t+1)}$ using (20).
Update price k, $\rho_k^{(t+1)}$ using (20) and timeshares $a_{j,k}^{(t)*}$ from links j, $j \neq i$.
Update $L_i(a_{i,k}, P_{s,k}^{(i)}, \alpha_{i,k}^{(t+1)}, {\gamma_i}^{(t+1)}, {\beta_i}^{(t+1)}, \rho_k^{(t+1)})$ (18).
Solve problem (19) for $(a_{i,k}^{(t+1)*}, P_{s,k}^{(i)(t+1)*})$.
Broadcast: tentative timeshares $a_{i,k}^{(t+1)*}$, $\forall k \in \Psi_K$.
RETURN $\tilde{\mathbf{T}}_{i,k}^{(t+1)} = \mathbf{T}_{i,k}(\mathbf{P}_k^{(i)(t+1)*})^{1/2}, \forall k \in \Psi_K$

Theorem 4: For a sufficiently small step size $\eta > 0$, Algorithm 1 converges to the globally optimal solution (Pareto-optimal NBS) of problem (11).

Proof: See Appendix B in [14]. □

It is worth noting that besides its optimality and distributed implementation, Algorithm 1 greatly reduces the computational time for large networks. Instead of dealing with $N(MK+K)$ variables in the centralized problem (11), Algorithm 1 involves $MK+K$ variables.

C. Distributed Bargaining Algorithm

The optimal solution of the relaxed problem tells which links wish to access which channels and for how long. In other words, the *preferences* of different links over the pool of available channels are revealed. In this section, we exclusively assign a channel to a link by considering preferences of all other links on that channel.

The gradients at the convergence point of Algorithm 1 must be zero if the globally optimal solution to (11) is an interior point of the feasible region. If the solution is a boundary point, the gradient at this point must be positive (negative) along the outward (inward) direction of the interior of the feasible region [16]. This fact is conveyed in (14) and (15) (the timeshare $a_{i,k} = 0$ iff $P_{s,k}^{(i)} = 0, \forall s = \{1, \ldots, M\}$).

Let Δ_i be the amount by which the allocated rate for link i (under timesharing) exceeds its demand c_i:

$$\Delta_i \overset{\text{def}}{=} \sum_{k\in\mathbf{S}_i}\left(a_{i,k}\sum_{s=1}^{M}\log(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}})\right) - c_i \tag{21}$$

When $P_{s,k}^{(i)} > 0$, (14) implies:

$$\frac{1}{g_{s,k}^{(i)}}\left(\alpha_{i,k}+\gamma_i\right)(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}}) = \frac{1}{\Delta_i} + \beta_i, \forall s = 1, \ldots, M. \tag{22}$$

Plugging $1/\Delta_i$ from (22) into (15) and after some manipulations, we get:

$$\frac{\partial L}{\partial a_{i,k}} = \begin{cases} F_{i,k} - \rho_k & \text{if } a_{i,k} > 0 \\ -\rho_k & \text{if } a_{i,k} = 0 \end{cases} \tag{23}$$

where

$$F_{i,k} \overset{\text{def}}{=} \left(\frac{1}{\Delta_i} + \beta_i\right)\sum_{s=1}^{M}\log\left(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}}\right) - \frac{\alpha_{k,i}+\gamma_i}{a_{i,k}}\sum_{s=1}^{M}P_{s,k}^{(i)}. \tag{24}$$

Recalling (15), (23) suggests that at the optimal solution, link i should exclusively occupy channel f_k if $F_{i,k} > \rho_k$; otherwise, link i should timeshare the channel with other links or not use it. ρ_k is interpreted as the price of using f_k, which is "flat" for all buyers/links. $F_{i,k}$ can be interpreted as the "payoff" that link i gets from "investing" on channel k. If channel f_k is exclusively allocated to one link, this link must have the highest $F_{i,k}$. This means the most efficient/needy user (of channel k) wins the channel. Formally, the following rule selects the optimal link for f_k:

$$a_{i',k} = \begin{cases} 1 & \text{if } i' = \arg\max_{\forall i \in \Phi_N} F_{i,k} \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

To execute the above rule in a distributed manner, each link i broadcasts a vector $\boldsymbol{F}_i \stackrel{\text{def}}{=} \{F_{i,1}, \ldots, F_{i,K}\}$. After receiving $\boldsymbol{F}_j$ from its neighbors, link i can autonomously determine the set of channels $\mathbf{A}_i$ it should select (when comparing $F_{i,k}$ of different links, if a tie happens, we randomly pick any of the links). Note that we assume secondary users are truthful and cooperative when broadcasting their "payoffs". Dealing with untruthful users is out of the scope of this work.

Economical Interpretation: Consider $F_{i,k}$ in (24). The first term is the weighted rate that link i can achieve from channel k. The second term is the weighted power that link i invests on channel k. Hence, the "payoff" $F_{i,k}$ is indeed the weighted rate that link i gets from channel k discounted by its allocated (weighted) power. For the same weighted power and the same scalar $(\frac{1}{\Delta_i} + \beta_i)$, the higher the channel gain $g_{s,k}^{(i)}$ of link i on channel k, the more likely that link i will win the channel. However, if two links have identical gains on channel k and the same weighted power, then the link with a smaller Δ_i (compared with its demand) is likely to win the channel. This fact ensures fair resource allocation.

After knowing its set of allocated channels A_i, it is necessary for link i to re-solve the power allocation problem to ensure optimality and QoS satisfaction, as follows:

$$\begin{aligned} &\underset{\{P_{s,k}^{(i)} \geq 0, \forall s=1,\ldots,M, \forall k \in \mathbf{A}_i\}}{\text{maximize}} \quad \sum_{k \in \mathbf{A}_i} \sum_{s=1}^{M} \log(1 + g_{s,k}^{(i)} P_{s,k}^{(i)}) \\ &\text{s.t.} \quad \sum_{k \in \mathbf{A}_i} \sum_{s=1}^{M} P_{s,k}^{(i)} \leq P_{\max} \\ &\qquad \sum_{s=1}^{M} P_{s,k}^{(i)} \leq P_{\text{mask}}^{*}(i, f_k), \quad \forall k \in \mathbf{A}_i. \end{aligned} \tag{26}$$

Problem (26) is convex and hence can be solved efficiently using standard methods. In fact, (26) belongs to the class of generalized water filling problems with multiple water levels (one at each channel), which can be solved efficiently with the algorithms in [18].

If the optimum solution to (26) does not meet the rate demand c_i, link i needs to inform others through a Reallocation Request message (RRM) and increases its bargain to compete for additional channels, i.e., raise its "payoff" vector $\boldsymbol{F}_i$ in (24). Since β_i is the price of violating the minimum rate constraint C3' in (8), it is intuitive to raise β_i by a sufficiently small step-size δ so that i wins only one additional channel at a time. δ and channel l that link i wants to acquire are found by Algorithm 2.

The idea of Algorithm 2 is to first find the vector of winning "payoffs" ($\boldsymbol{F}_{\max}$) for all channels, and then see how far the "payoff" vector $\boldsymbol{F}_i$ of link i is from these values (vector $\boldsymbol{\Theta}_i$). Recalling (24), if link i wants to win channel k that is currently not allocated for i, then δ must be set to be strictly greater than $\frac{\Theta_{i,k}}{\Upsilon_{i,k}}$. However, link i wants to request only one channel at a time. For that, we sort the elements of $\boldsymbol{\Theta}_i$ in an ascending order, then set δ to be the average of the two smallest positive elements of $\boldsymbol{\Theta}_i$.

Using its updated price, $\beta_i = \beta_i + \delta$, link i recalculates the "payoff" vector $\boldsymbol{F}_i$. Consequently, it broadcasts a RRM, containing channel l and the updated $\boldsymbol{F}_i$. Upon hearing the message, all links record the new $\boldsymbol{F}_i$. Then, the current "owner" (link j) of channel l excludes l from its set of allocated channels $\mathbf{A}_j$ (since link i is now more "competitive"). Both links i and j re-solve the power allocation problem (26) and check if their demands are met. The process of increasing the bidding price to bargain for additional channels continues until all links get their requested rates.

Algorithm 2 Find increment δ for the price of violating link i's rate demand (problem (6)) and channel l that i is about to acquire:

Input: $\boldsymbol{F}_i, \forall i \in \Phi_N$
Output: δ and l
$\Upsilon_{i,k} \stackrel{\text{def}}{=} \sum_{s=1}^{M} \log(1 + \frac{g_{s,k}^{(i)} P_{s,k}^{(i)}}{a_{i,k}})$
$\boldsymbol{F}_{\max} \stackrel{\text{def}}{=} \{F_{\max}(1), \ldots, F_{\max}(K)\}$ where $F_{\max}(k) = \max\{F_{i,k}\}, \forall i \in \Phi_N$.
$\boldsymbol{\Theta}_i \stackrel{\text{def}}{=} \{\Theta_{i,1}, \ldots, \Theta_{i,K}\}$ with $\Theta_{i,k} = F_{\max}(k) - F_{i,k}$.
Sort $\boldsymbol{Z}_i \stackrel{\text{def}}{=} Sort(\boldsymbol{\Theta}_i)$ in ascending order.
Let $Z_i(m)$ is the smallest positive element in $\boldsymbol{Z}_i$.
Set: $\delta = \frac{(Z_i(m) + Z_i(m+1))}{2}$.
Channel that link i is going to acquire is the index of $Z_i(m)$ in $\boldsymbol{\Theta}_i$ before sorting.
RETURN: δ and channel index l.

We assume that there is enough spectrum in the network to meet the minimum demands of all links (necessary condition to apply NBS [2]), so that problem (6) is feasible. This can be easily realized through an admission/congestion control mechanism. Hence, the bargaining process eventually stops. If no RRM is heard for a given time duration (set as $Timer$), all links start transmitting on their selected channels. The channel and power allocation for problem (6) is summarized in Algorithm 3.

Algorithm 3 Distributed Bargaining Algorithm to Design Precoders and Allocate Channels for Node i at Time $(t+1)$:

Execute Algorithm 1 (until convergence)
Payoff vector computation $\boldsymbol{F}_i$ (using (24))
Enter channel allocation phase:
Link i broadcasts its payoff vector $\boldsymbol{F}_i$. Then, sets *Timer*
while $Timer$ not expired **do**
  Upon receiving $\boldsymbol{F}_j$ from neighbors, update the set of allocated channels $\mathbf{A}_i$ using (25).
  Execute the power allocation (26) and check if $R_i \geq c_i$
  if $R_i < c_i$ **then**
    Compute δ and the channel index l
    Set $\beta_i = \beta_i + \delta$ and update $\boldsymbol{F}_i$ using (24) to acquire (additional) channel l
    Broadcast the new $\boldsymbol{F}_i$, RRM and reset *Timer*
  end if
  If a RRM is heard, reset *Timer*
end while
RETURN $\tilde{\mathbf{T}}_{i,k}^{(t+1)} = \mathbf{T}_{i,k}(\mathbf{P}_k^{(i)(t+1)*})^{1/2}, \forall k \in \mathbf{A}_i$

IV. Numerical Results

We simulate a CMIMO network in which each node is equipped with 4 antennas. The total number of channels is 20, each with bandwidth of 16 MHz. The number of links is varied from 3 to 10. We set $P_{\max} = 1$ W and $P_{\text{mask}}^{*}(f_k) = 0.5$ W $\forall f_k$. Noise floor plus PUs interference is -100 dBm/Hz. Without loss of generality, we set the rate demands of each link to 2 bits/s/Hz. Simulation results are averaged over 10 runs. In each run, CMIMO nodes are randomly distributed on a square field of length 100 m. Channels are assumed to be stationary during each simulation experiment, with a free-space attenuation factor of 2. The spreading angles of arrival signals vary from $-\pi/5$ to $\pi/5$. Following a similar approach to Algorithms 1 and 3, we develop algorithms to maximize the CMIMO network throughput, called NET-MAX (see Section V in [14]), which serves as a performance benchmark (in terms of throughput).

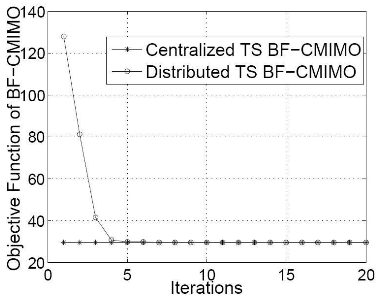

Fig. 2. Convergence of the distributed algorithm under timesharing (TS) for BF-CMIMO.

To evaluate the optimality and convergence of Algorithm 1 under timesharing (TS), we consider a network of 10 links. Spectrum heterogeneity is captured by making channels ith, $i+1$, $i+2$ are not available for link i. Figure 2 depicts the dual function (17) vs. iterations. Algorithm 1 of TS BF-CMIMO converges to the optimal centralized solution after 5 iterations. Under exclusive channel allocation (no TS), we observed that Algorithm 3 for BF-CMIMO often needs less than 3 additional iterations to reallocate channels (figure not shown for brevity).

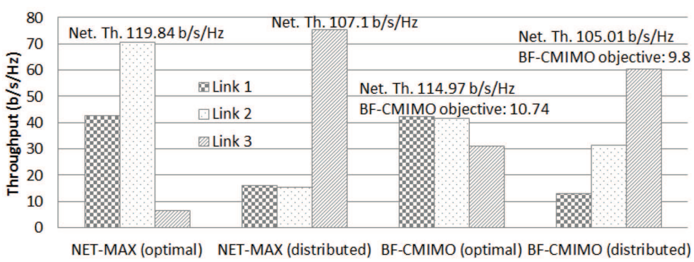

Fig. 3. Distributed BF-CMIMO and NET-MAX algorithms vs. optimal solutions (via exhaustive search).

To compare the performance of the heuristic algorithm BF-CMIMO with its optimal solution under the exclusive channel occupancy policy, we run an exhaustive search on a small network of 3 links and 10 channels. Figure 3 shows that the value of the objective of BF-CMIMO under Algorithm 3 is 9.8, compared with the optimal value of 10.74. This suggests that Algorithm 3 achieves 93% of the optimal solution. This also shows that the throughput of the distributed BF-CMIMO algorithm (105.01 bits/s/Hz) is about 9% less than that of the optimal NET-MAX solution (119.84 bits/s/Hz).

Figure 4 shows Jain's fairness index of the Algorithms 1 and 3 of BF-CMIMO and the corresponding algorithms for NET-MAX. Algorithms that rely on NB (with or without TS) achieve significantly better fairness than those of NET-MAX. As the number of links increases, the fairness index under NET-MAX (with or without TS) decreases. However, BF-CMIMO algorithms maintain quite stable fairness for different network sizes. This is because under BF-CMIMO, channels (or their timeshares) are allocated while accounting for the amount of extra rate Δ_i. Jain's index for the distributed algorithm under BF-CMIMO with

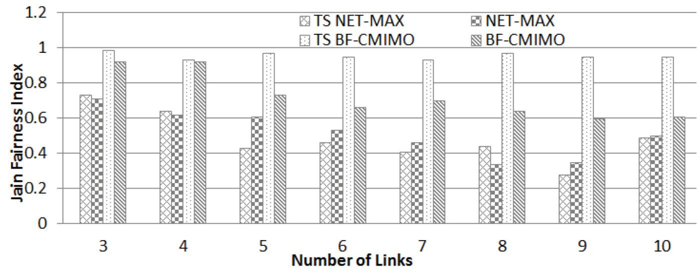

Fig. 4. Jain's fairness index under BF-CMIMO and NET-MAX, with and without TS.

exclusive channel allocation is about 19% less than that under TS.

V. Conclusions

In this paper, we developed fully distributed algorithms to jointly allocate channels (under the exclusive channel occupancy), and optimize power allocation and antenna patterns (through precoding matrices) for cognitive MIMO networks. The proposed algorithms allow cognitive MIMO links to propose their rate demands, cooperate and bargain to get their channel assignment, and optimize their precoders under the Nash Bargaining framework.

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