Game-theoretic Quorum-based Frequency Hopping for Anti-jamming Rendezvous in DSA Networks

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***Abstract*—Establishing communications in a dynamic spectrum access (DSA) network requires the communicating parties to "rendezvous" before transmitting their data packets. Frequency hopping (FH) is an effective rendezvous method that does not rely on a predetermined control channel. Recently, "quorum-based" FH approaches have been proposed for asynchronous rendezvous in DSA networks. These approaches are highly vulnerable to jamming, especially when the attacker is an insider node (i.e., a compromised node). In this paper, we investigate the problem of two secondary users (SUs), a transmitter and a receiver, try to rendezvous in the presence of a third SU acting as a jammer. The jammer is aware of the underlying (quorum-based) rendezvous design. First, we consider the case when all SUs are time-synchronized and are aware of the "rendezvous channel". We formulate the problem as a three-player game between the transmitter, receiver, and jammer. The transmitter and receiver try to maximize the number of successful rendezvous slots, while minimizing the number of jammed rendezvous slots. The jammer has the opposite objective. We show that this game does not have a pure Nash equilibrium (NE). Accordingly, we formulate a simplified two-player game between the receiver and jammer (assuming a uniform strategy by the transmitter), and derive multiple pure NE strategies. Next, we study the case when the rendezvous channel is unknown and obtain the Bayesian NE. Finally, the asynchronous case is addressed by exploiting the "rotation closure property" of quorum systems. Our numerical experiments show that uncertainty about the transmitter's strategy improves the anti-jamming rendezvous performance. They also show that the rendezvous performance improves if the receiver and jammer are time-synchronized, and also improves if the receiver and jammer have a common guess about the transmitter's strategy.**

***Keywords*—Bayesian games, frequency hopping, quorum systems, rendezvous, three-player games.**

I. Introduction

Motivated by the need for more efficient utilization of the licensed spectrum, and supported by recent regulatory policies (e.g., [9]), significant research has been conducted towards developing cognitive radio (CR) technologies for dynamic spectrum access (DSA) networks. CR devices utilize the available spectrum in a dynamic and opportunistic fashion without interfering with co-located *primary users* (PUs). The communicating entities of an opportunistic CR network are called *secondary users* (SUs).

Establishing a communication link in a DSA network requires nodes to rendezvous on a common channel for the purpose of exchanging critical information, such as transmission parameters, topology changes, etc. In the absence of centralized control, the rendezvous process needs to be carried out in a distributed manner. Many existing MAC protocols for DSA networks rely on a dedicated global or group control channel[1] for rendezvous (e.g., [8], [14], [19]). Presuming a common control channel (CCC) surely simplifies the rendezvous process. However, it creates a network bottleneck. Alternatively, frequency hopping (FH) provides an effective method for rendezvousing without relying on a predetermined CCC [12], [17], [22].

Recently, several "quorum-based" FH schemes have been proposed for asynchronous rendezvous in DSA networks (see, for example, [1], [3]–[7], [18], [20], [21] and references therein). Quorums have been previously used in distributed systems to solve the mutual exclusion problem, the agreement problem, and the replica control problem [13]. One key advantage of quorum-based FH designs is their robustness to synchronization errors [16]. Specifically, some quorum systems (e.g., grid, torus, and cyclic quorum systems [15]) enjoy the "rotation closure property," whereby any two quorums in such a quorum system overlap even if they are cyclically rotated by an arbitrary amount. This property allows these quorum systems to be used for asynchronous operation.

On the other hand, quorum-based FH schemes are highly vulnerable to jamming attacks. This is because of the systematic nature of their design, which makes them exposed to adversaries. These schemes become more vulnerable when an attack is launched by an insider SU, e.g., when a trusted SU has been compromised and its secrets have been partially or completely revealed to the attacker.

In this paper, we consider an attack model in which the attacker is aware of the quorum-based FH design used by various SUs to rendezvous. The goal of the attack is to prevent SUs from rendezvousing, by maximizing the number of jammed rendezvous instances while minimizing the number of successful rendezvous instances. Figure 1 shows a transmitter-receiver link in a DSA network that operates in the presence of a jammer. All SUs (i.e., transmitter, receiver, and jammer) operate using a grid-quorum-based FH approach. Each FH sequence is divided into frames (in our example, the frame length is nine slots). The slots of a frame are arranged into a square grid (a 3×3 grid in our example). Each SU selects a column and a row from the grid. The slots that correspond to the selected column and row are assigned a channel called the *rendezvous channel* (channel f in Figure 1), and the remaining

[1]In this paper, we use the terms channel and frequency interchangeably.

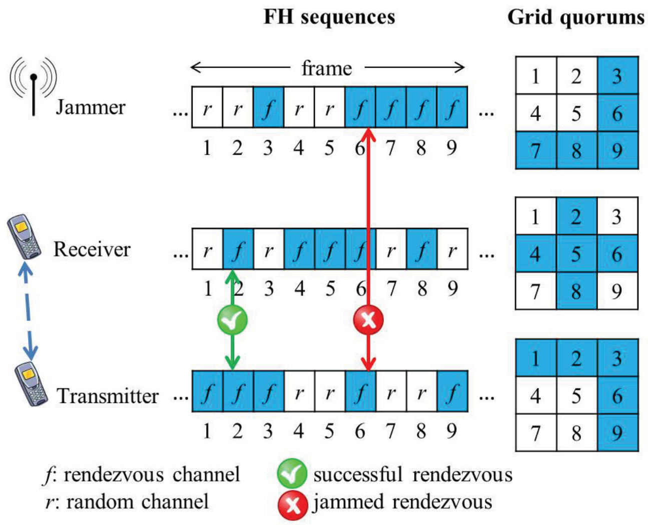


Fig. 1: Two nodes try to rendezvous in the presence of a jammer. All nodes follow a grid-quorum-based FH approach.

slots are assigned a random channel (denoted by r in the figure). Note that $r \neq f$. A successful rendezvous instance refers to a time slot where the transmitter and receiver are tuned to channel f, while the jammer is tuned to a different channel. If the transmitter, receiver, and jammer are tuned to a common channel during the same slot, then this slot is considered as a *jammed rendezvous slot*.

Main Contributions–The main contributions of this paper are as follows:

- We first consider the case when the rendezvous channel is known to all nodes, including the attacker. We formulate the time-synchronous rendezvous problem as a three-player matrix game, played by the rendezvousing transmitter, the receiver, and the jammer. We show that this game does not have a pure Nash equilibrium (NE). Accordingly, we formulate a simplified two-player matrix game between the receiver and the jammer, assuming that the transmitter follows a uniformly random strategy. In such a formulation, we assume that only the receiver is aware of the presence of the jammer, but not the transmitter. We derive multiple pure NE strategies for this game.
- We then formulate the two-player time-synchronous rendezvous game when the rendezvous channel is unknown as a Bayesian game. The Bayesian NE for this game is derived.
- Using the rotation closure property of grid quorums, the two-player asynchronous rendezvous problem is then mapped into the synchronous rendezvous game.

Paper Organization–In Section II, we present the system and adversarial models. We briefly explain the grid-quorum-based FH rendezvous design in Section III. In Section IV, we introduce the three-player game for the synchronous anti-jamming rendezvous problem with known rendezvous channel and show the nonexistence of any pure-strategy NE. The simplified two-player receiver/jammer synchronous game formulation is provided in Section V. The Bayesian game formulation for the synchronous rendezvous problem with an unknown rendezvous channel is explained in Section VI. The asynchronous rendezvous problem is presented in Section VII. Numerical results are discussed in Section VIII. In Section IX, we provide directions for future research, followed by the concluding remarks in Section X.

II. System and Adversarial Models

A. System Model

We consider an SU link (a transmitter and a receiver) in an ad hoc DSA network, operating over a set $\mathcal{L} = \{1, 2, \dots, L\}$ of channels in the presence of an adversary. The transmitter and receiver can successfully communicate over a channel if this channel is not jammed. They rendezvous using a quorum-based FH approach similar to the one in [2]. Without loss of generality, we assume that FH occurs on a per-slot basis, with a slot duration of T seconds. A packet can be exchanged between two nodes if they hop onto the same channel during the same time slot. As in previous quorum-based FH designs, in our setup each FH sequence is divided into several time frames. Each frame corresponds to a block of time-frequency pairs.

B. Adversarial Model

We consider a time-slotted jammer, with a slot duration of T seconds. In each time slot, the jammer injects an interfering signal on one of the channels in $\mathcal{L}$, with the purpose of preventing the receiver from correctly decoding the transmitter's message. The jamming attack is carried out by a compromised node (i.e., insider attack), so the attacker is aware of the systematic quorum-based FH approach used by network nodes to rendezvous. However, the attacker does not know the specific quorums used by the transmitter and receiver.

In the following section, we review the nested grid-quorum-based approach proposed in [2]. Then, we explain what salient features of this approach are exposed to the jammer.

III. Nested Grid-quorum-based FH Rendezvous

A. Preliminaries

Before explaining the nested quorum design proposed in [2], we define some terminology related to quorum systems.

Definition 1: Given a set $U = \mathbb{Z}_n = \{0, 1, \dots, n-1\}$, a quorum system Q under U is a collection of non-empty subsets of U, each called a quorum, such that:

$$\forall G, H \in Q : G \cap H \neq \emptyset. \tag{1}$$

Throughout the paper, $\mathbb{Z}_n$ indicates the set of non-negative integers that are less than n.

Definition 2: Given a non-negative integer i and a quorum G in a quorum system Q under $\mathbb{Z}_n$, we define the operation *rotate*$(G, i) = \{(x+i) \mod n, x \in G\}$ to denote a cyclic rotation of quorum G by i times.

Definition 3: A quorum system Q under $\mathbb{Z}_n$ is said to satisfy *the rotation closure property* if:

$$\forall G, H \in Q, i \in \{0, 1, \ldots, n-1\} : G \cap rotate(H, i) \neq \emptyset. \quad (2)$$

The rotation closure property is what makes quorum systems suitable for operating in asynchronous FH settings [16].

As discussed in [15], concerning the "neighbor sensibility" in mobile ad hoc networks, quorum systems are typically characterized by two metrics: the smallest quorum overlap size (SQOS) and the maximum quorum overlap separation (MQOS). The SQOS of a quorum system Q is defined as the smallest number of overlapping elements between any two quorums in Q. The MQOS of a quorum system Q is defined as the maximum separation between any two overlapping elements of any two quorums in Q. MQOS is formally defined as follows:

Definition 4: For a quorum system Q under $\mathbb{Z}_n$, MQOS is given by:

$$\max_{\substack{i,j \in G \cap H \\ G,H \in Q}} \left\{ \min \left\{ (i-j) \mod n, (j-i) \mod n \right\} \right\}. \quad (3)$$

For example, if G and H represent the transmitter's and receiver's quorums in Figure 1, then they overlap in slots 2 and 6. In this case, the maximum separation is 4. To obtain MQOS, this computation will need to be done for all quorum pairs in Q.

In a quorum-based rendezvous design, the SQOS and MQOS metrics can also be used to characterize the robustness of the quorum system to jamming attacks. As mentioned in [15], the grid quorum system has the smallest MQOS and the largest SQOS among the quorum systems that were considered in [15]. In fact, grid quorum systems achieve the smallest worst-case rendezvous delay in the presence of a jammer. Note that the rendezvous time is different for different pairs of quorums, and hence we consider the worst-case delay for a given quorum system. Accordingly, we formulate the games in this paper using a grid quorum system. We note here that a similar approach can be used to analyze other quorum systems. Next, we formally define the grid quorum system.

Definition 5: A grid quorum system arranges the elements of $\mathbb{Z}_n$, $n = q^2$ for some positive integer q, as a $q \times q$ array. A quorum is formed from the elements of one column and one row of the grid (see Figure 1).

The grid quorum system satisfies the rotation closure property [16]. Figure 2 illustrates this property for two quorums G and H in a grid quorum system under $\mathbb{Z}_{16}$. The two quorums have at least two intersections (labeled I in Figure 2). The shifted quorums $G' = rotate(G, 1)$ and $H' = rotate(H, 2)$ intersect at the two elements labeled as I'.

B. Nested Grid-quorum-based FH Rendezvous Algorithm (NGQFH)

To simplify the exposition, we first explain a non-nested version of the grid-quorum rendezvous algorithm. Time is divided into frames, each containing m slots (m needs to be the square of a positive integer). The slots of each frame are

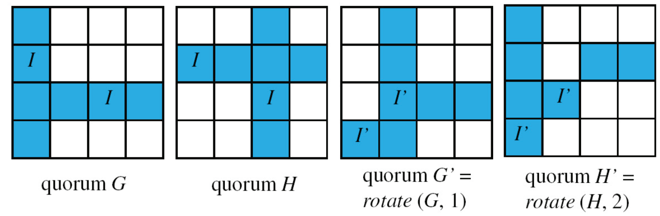


Fig. 2: Rotation closure property in grid quorum systems.

formed as a $\sqrt{m} \times \sqrt{m}$ grid, from which the quorums are derived. For each FH sequence, a grid quorum (a column and a row) is randomly selected. Given a set of available channels, one common channel is assigned to all quorum slots (henceforth, called the rendezvous channel).

In the (more general) nested rendezvous algorithm, every frame of every FH sequence uses $\sqrt{m}-1$ rendezvous channels [2]. The number of rendezvous channels is called the *nesting degree*. Again, a $\sqrt{m} \times \sqrt{m}$ quorum is selected for each FH sequence, and the first rendezvous channel is assigned to the slots that correspond to the selected quorum. This $\sqrt{m} \times \sqrt{m}$ quorum is called the *outer-most quorum*. The column and row that correspond to the outer-most quorum are then deleted from the grid, and another quorum is selected from the remaining $(\sqrt{m}-1) \times (\sqrt{m}-1)$ grid. A second rendezvous channel is then assigned to this smaller quorum. This procedure continues for $\sqrt{m}-1$ iterations.

To explain, we consider an example with $m = 9$ (hence, each frame contains $\sqrt{m} - 1 = 2$ rendezvous channels). Consider one frame of an FH sequence. Sequence construction proceeds as follows:

1. Construct a grid quorum system Q under $\mathbb{Z}_9$. Q has 9 different quorums, each containing $2\sqrt{9} - 1 = 5$ elements that comprise one row and one column of the 3×3 grid.
2. Select the outer-most quorum (denoted by G_1) from Q (e.g., $G_1 = \{1, 3, 4, 5, 7\}$, where each element in G_1 represents the index of a time slot in a 9-slot frame).
3. Assign the first rendezvous channel (denoted by f_1) to the slots that correspond to G_1.
4. Delete quorum G_1 from the original 3×3 grid and select the next outer-most quorum (denoted by G_2) from the resulting 2×2 grid (e.g., $G_2 = \{2, 6, 8\}$). Then, assign another rendezvous channel (denoted by f_2) to the slots that correspond to G_2.
5. Assign a random channel (denoted by f_x) to each of the remaining unassigned (non-quorum) slots in the frame.

The above procedure is repeated for all the frames in the FH sequence. Figure 3 shows the resulting frames for the transmitter and receiver FH sequences.

The above nested grid-quorum-based algorithm is executed at the two rendezvousing nodes. This design enables nodes to potentially rendezvous on multiple channels during each frame. If the jammer restricts its attack to one channel during the whole frame, it will not be able to prevent the nodes from rendezvousing, since they can meet on the remaining $\sqrt{m}-2$ rendezvous channels. Recall from Section II-B that the jammer can only jam one channel per time slot. Hence, the jammer

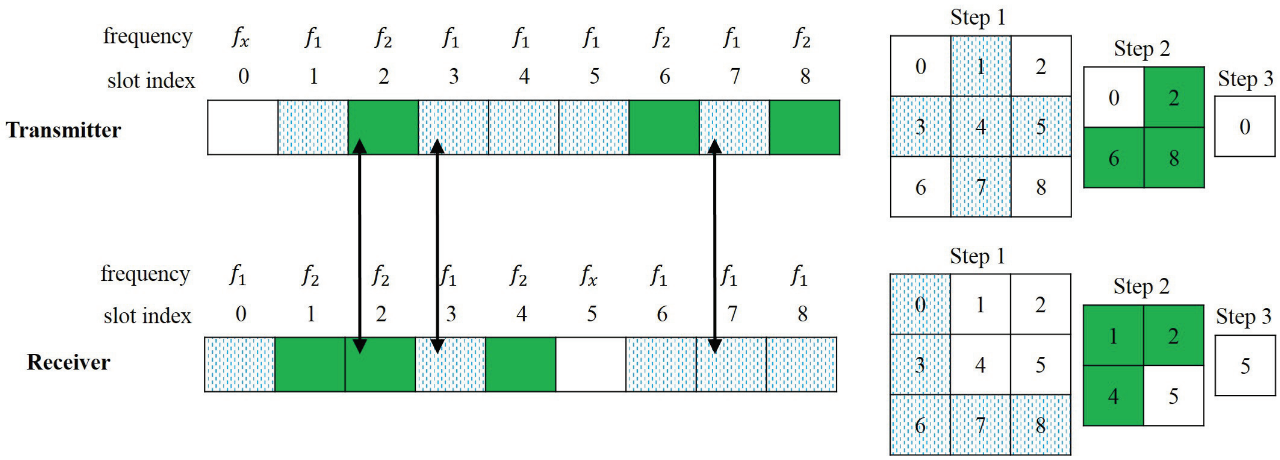


Fig. 3: Example of the nested grid-quorum-based rendezvous algorithm when $m = 9$. This quorum-based design is symmetric, i.e., the transmitter sequence can be used by the receiver, and vice versa.

needs to select the appropriate $\sqrt{m}-1$ nested quorums and jam accordingly.

Remark 1: If the transmitter and receiver adopt a quorum-based FH design, then it is in the jammer's interest to follow the same design for two reasons. First, using this approach, the jammer ensures overlapping with the transmitter and receiver separately, and if he selects the quorum appropriately he can overlap with both simultaneously (i.e., block the rendezvous slot). Second, given that the jammer is not synchronized with the transmitter and receiver, the rotation closure property ensures that the jammer can overlap with each node, hence facilitating his jamming effort.

As a first step, we restrict the analysis in this paper to grid quorums with a nesting degree of one. Our analytical approach can be extended to higher nesting degrees by formulating our problem as *a matrix game with games as components* [10]. In Section IX, we explain how to extend our analysis to the case when the nesting degree is greater than one.

IV. Three-player Synchronous Rendezvous Game on a Known Rendezvous Channel

The anti-jamming rendezvous problem can be stated as follows. Two SUs, a transmitter and a receiver, are trying to rendezvous while a third SU, a jammer, tries to prevent them from rendezvousing. The jammer knows that the transmitter and receiver follow a nested grid-quorum-based FH approach with a frame length of m. Moreover, he knows the rendezvous channel. However, the jammer does not know which quorums are selected by the transmitter and receiver. If the jammer randomly selects a quorum to follow, it will overlap with either the transmitter or the receiver, but not necessarily with both. The number of different transmitter-receiver quorum selections is given by:

$$K_m = \left(\prod_{i=0}^{\sqrt{m}-2} (\sqrt{m}-i)^2 \right)^2 . \quad (4)$$

We use game theory to capture the interactions between the transmitter (T), receiver (R), and jammer (J). A game is characterized by a set of players, a set of actions (strategies) for each player, and a payoff (utility) function for each player. The actions (strategies) that can be taken by each of the three players are the m different $\sqrt{m}\times\sqrt{m}$ grid quorums. We use s_T, s_R, and s_J to denote the strategies of T, R, and J, respectively. All players have the same strategy space, denoted by S, which consists of all quorums (pure strategies). Furthermore, let $s_T \stackrel{\text{def}}{=} (s_{T,r}, s_{T,c})$, $s_R \stackrel{\text{def}}{=} (s_{R,r}, s_{R,c})$, and $s_J \stackrel{\text{def}}{=} (s_{J,r}, s_{J,c})$, where $s_{T,r}, s_{R,r}, s_{J,r} \in S_r = \{1, 2, \ldots, \sqrt{m}\}$ represent the row indices of the T, R, and J grid quorums, respectively, and $s_{T,c}, s_{R,c}, s_{J,c} \in S_c = \{1, 2, \ldots, \sqrt{m}\}$ represent their column indices, respectively. The utility functions for T, R, and J are denoted by u_T, u_R, and u_J, respectively. We let $u_T = u_R = -u_J$, and define u_T as follows:

$$\begin{aligned} u_T(s_T, s_R, s_J) &= \text{\# of unjammed rendezvous slots per frame} \\ &\quad - \text{\# of jammed rendezvous slots per frame.} \end{aligned} \quad (5)$$

Remark 2: Another possible utility function for T would be the number of successful (unjammed) rendezvous slots, i.e., without discounting the number of jammed rendezvous slots. Under this utility, the game has a pure-strategy NE, given by $s_T = s_R = s_J$. However, this NE is inefficient, as it leads to $u_T = u_R = 0$, i.e., T and R do not successfully rendezvous in any slot. Furthermore, this utility does not account for the energy loss at T during the unsuccessful rendezvous slots. Our preliminary investigation of the three-player game under this utility shows that even though there are multiple NEs, the players do not always converge to any of them. Specifically, if they repeatedly play a best-response strategy and follow a parallel update procedure, they may or may not converge to a NE, depending on their initial strategies.

Since the frame length m is typically small and the strategy space is discrete, we can model the above problem as a *matrix game*. Figure 4 depicts the matrix game for the case of $m = 4$, assuming all players are synchronized. Each cell in Figure 4 includes the utility values u_R, u_J, and u_T at the given strategies. As shown in the figure, the game does not

Tx

Tx = (1, 1)

Rx \ Jammer	(1, 1)	(1, 2)	(2, 1)	(2, 2)
(1, 1)	-3, 3 -3	-1, 1 -1	-1, 1 -1	-1, 1 -1
(1, 2)	-2, 2 -2	-2, 2 -2	0, 0 0	0, 0 0
(2, 1)	-2, 2 -2	0, 0 0	-2, 2 -2	0, 0 0
(2, 2)	-2, 2 -2	0, 0 0	0, 0 0	-2, 2 -2

Tx = (1, 2)

Rx \ Jammer	(1, 1)	(1, 2)	(2, 1)	(2, 2)
(1, 1)	-2, 2 -2	-2, 2 -2	0, 0 0	0, 0 0
(1, 2)	-1, 1 -1	-3, 3 -3	-1, 1 -1	-1, 1 -1
(2, 1)	0, 0 0	-2, 2 -2	-2, 2 -2	0, 0 0
(2, 2)	0, 0 0	-2, 2 -2	0, 0 0	-2, 2 -2

Tx = (2, 1)

Rx \ Jammer	(1, 1)	(1, 2)	(2, 1)	(2, 2)
(1, 1)	-2, 2 -2	0, 0 0	-2, 2 -2	0, 0 0
(1, 2)	0, 0 0	-2, 2 -2	-2, 2 -2	0, 0 0
(2, 1)	-1, 1 -1	-1, 1 -1	-3, 3 -3	-1, 1 -1
(2, 2)	0, 0 0	0, 0 0	-2, 2 -2	-2, 2 -2

Tx = (2, 2)

Rx \ Jammer	(1, 1)	(1, 2)	(2, 1)	(2, 2)
(1, 1)	-2, 2 -2	0, 0 0	0, 0 0	-2, 2 -2
(1, 2)	0, 0 0	-2, 2 -2	0, 0 0	-2, 2 -2
(2, 1)	0, 0 0	0, 0 0	-2, 2 -2	-2, 2 -2
(2, 2)	-1, 1 -1	-1, 1 -1	-1, 1 -1	-3, 3 -3

(row #, column #)

u_R, u_J
u_T

Fig. 4: Three-player matrix game for the 2×2 grid-quorum system ($m = 4$).

have any pure-strategy NE.

The transmitter utility for the 2×2 grid-quorum game can be written as:

$$u_T(s_T, s_R, s_J) = \begin{cases} -3, & \text{if } s_T = s_R = s_J \\ -2, & \text{if } s_R = s_J \neq s_T \\ & \text{or } s_T = s_J \neq s_R \\ -1, & \text{if } s_T = s_R \neq s_J \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

In general, the transmitter utility for the $\sqrt{m} \times \sqrt{m}$ grid-quorum game is given by (7).

Lemma 1: In the above three-player game, if a strategy profile (s_T, s_R, s_J) satisfies the following two conditions:

$$u_R(s_T, s_R, s_J) \geq u_R(s_T, s'_R, s_J), \forall s'_R \in S, s'_R \neq s_R \quad (8)$$

$$u_J(s_T, s_R, s_J) \geq u_J(s_T, s_R, s'_J), \forall s'_J \in S, s'_J \neq s_J \quad (9)$$

then there exists a transmitter strategy s^*_T, such that $u_T(s^*_T, s_R, s_J) > u_T(s_T, s_R, s_J)$.

Proof: From (7), it can be shown that a strategy profile (s_T, s_R, s_J) satisfies (8) and (9) if and only if $s_{J,r} = s_{T,r}, s_{J,c} = s_{T,c}, s_{R,r} \neq s_{T,r}$, and $s_{R,c} \neq s_{T,c}$. In this case, $u_T = -2$. If T deviates from its strategy, then the resulting strategy profile will be one of the following (in here, $A \neq B \neq C$ means $A \neq B$ and $B \neq C$):

1. $s_{R,r} \neq s_{T,r} \neq s_{J,r}$ and $s_{R,c} \neq s_{T,c} \neq s_{J,c}$
2. $s_{R,r} = s_{T,r} \neq s_{J,r}$ and $s_{R,c} \neq s_{T,c} \neq s_{J,c}$
3. $s_{R,r} \neq s_{T,r} \neq s_{J,r}$ and $s_{R,c} = s_{T,c} \neq s_{J,c}$
4. $s_{R,r} = s_{T,r} \neq s_{J,r}$ and $s_{R,c} = s_{J,c} \neq s_{T,c}$
5. $s_{R,r} = s_{J,r} \neq s_{T,r}$ and $s_{R,c} = s_{T,c} \neq s_{J,c}$
6. $s_{R,r} = s_{T,r} \neq s_{J,r}$ and $s_{R,c} = s_{T,c} \neq s_{J,c}$
7. $s_{R,r} = s_{J,r} \neq s_{T,r}$ and $s_{R,c} \neq s_{T,c} \neq s_{J,c}$
8. $s_{R,r} \neq s_{T,r} \neq s_{J,r}$ and $s_{R,c} = s_{J,c} \neq s_{T,c}$.

u_T for case 1 is 2, for cases 2 to 5 is $\sqrt{m} - 2$, for case 6 is $2\sqrt{m} - 5$, and for cases 7 and 8 is 0. So, in all the above eight cases, $u_T > -2$, and the lemma holds. ∎

Theorem 1: The three-player synchronous rendezvous game does not have a pure-strategy NE.

Proof: If there is a NE for the three-player rendezvous game, then it will be one of the strategy profiles defined by (8) and (9). However, from Lemma 1, none of the strategy profiles defined by (8) and (9) constitutes a NE. Therefore, there is no pure-strategy NE for the above three-player game. ∎

Remark 3: Note that we do not consider mixed NE strategies in our rendezvous game. Selecting quorums according to a mixed strategy makes the rotation closure property (see Definition 3) of grid quorum systems inapplicable. In order to apply the rotation closure property, the outer-most quorum of the FH sequence (see Figure 3) needs to be the same in all successive frames. Recall that the rotation closure property is what makes quorum systems suitable for operating in asynchronous FH settings.

V. Two-player Synchronous Rendezvous Game on a Known Rendezvous Channel

As shown in Theorem 1, the three-player rendezvous game does not have a NE. In this section, we formulate a simplified two-player matrix game between R and J, assuming that T follows a uniformly random strategy. In this formulation, we assume that only R is aware of the presence of the jammer, but not T. Each T strategy results in a different two-player game, that have multiple pure NE strategies, as will be shown in this section. Since T selects any strategy in S with probability $1/|S|$ (where $|S|$ is the cardinality of S), R and J uniformly randomize between the NE strategies that correspond to the different T strategies. First, we will study this game for the special cases of 2×2 and 3×3 grid quorums. Then, we will consider the general case of $\sqrt{m} \times \sqrt{m}$ grid quorum.

A. The 2×2 Grid-quorum Game

Each grid in Figure 4 corresponds to a two-player game between R and J for a fixed s_T. Since $m = 4$, T has four

$$
u_T(s_T, s_R, s_J) = \begin{cases} 1-2\sqrt{m}, & \text{if } s_T = s_R = s_J \\ -\sqrt{m}, & \text{if } s_{T,r} = s_{R,r} = s_{J,r} \text{ and } s_{R,c} = s_{J,c} \neq s_{T,c}, \text{ or } s_{R,r} = s_{J,r} \neq s_{T,r} \text{ and } s_{T,c} = s_{R,c} = s_{J,c} \\ & \text{or } s_{T,r} = s_{R,r} = s_{J,r} \text{ and } s_{T,c} = s_{J,c} \neq s_{R,c}, \text{ or } s_{T,r} = s_{J,r} \neq s_{R,r} \text{ and } s_{T,c} = s_{R,c} = s_{J,c} \\ & \text{or } s_{T,r} = s_{R,r} = s_{J,r} \text{ and } s_{R,c} \neq s_{T,c} \neq s_{J,c}, \text{ or } s_{R,r} \neq s_{T,r} \neq s_{J,r} \text{ and } s_{T,c} = s_{R,c} = s_{J,c} \\ -2, & \text{if } s_{T,r} = s_{J,r} \neq s_{R,r} \text{ and } s_{T,c} = s_{J,c} \neq s_{R,c}, \text{ or } s_{R,r} = s_{J,r} \neq s_{T,r} \text{ and } s_{R,c} = s_{J,c} \neq s_{T,c} \\ -1, & \text{if } s_{T,r} = s_{R,r} = s_{J,r} \text{ and } s_{R,c} = s_{T,c} \neq s_{J,c}, \text{ or } s_{R,r} = s_{T,r} \neq s_{J,r} \text{ and } s_{T,c} = s_{R,c} = s_{J,c} \\ 2, & \text{if } s_{R,r} \neq s_{T,r} \neq s_{J,r} \text{ and } s_{R,c} \neq s_{T,c} \neq s_{J,c} \\ \sqrt{m}-2, & \text{if } s_{R,r} = s_{T,r} \neq s_{J,r} \text{ and } s_{R,c} \neq s_{T,c} \neq s_{J,c}, \text{ or } s_{R,r} \neq s_{T,r} \neq s_{J,r} \text{ and } s_{R,c} = s_{T,c} \neq s_{J,c} \\ & \text{or } s_{R,r} = s_{T,r} \neq s_{J,r} \text{ and } s_{R,c} \neq s_{T,c} = s_{J,c}, \text{ or } s_{R,r} \neq s_{T,r} = s_{J,r} \text{ and } s_{R,c} = s_{T,c} \neq s_{J,c} \\ & \text{or } s_{R,r} = s_{T,r} \neq s_{J,r} \text{ and } s_{R,c} = s_{J,c} \neq s_{T,c}, \text{ or } s_{R,r} = s_{J,r} \neq s_{T,r} \text{ and } s_{R,c} = s_{T,c} \neq s_{J,c} \\ 2\sqrt{m}-5, & \text{if } s_{R,r} = s_{T,r} \neq s_{J,r} \text{ and } s_{R,c} = s_{T,c} \neq s_{J,c} \\ 0, & \text{otherwise.} \end{cases} \tag{7}
$$

strategies, and hence the four grids. Each game has three pure NE strategies. The utility function of this game is given by (6). For a given s_T, any pair of R/J strategies (s_R, s_J) constitutes a NE if and only if $s_J = s_T \neq s_R$. All NEs result in $u_T = -2$.

B. The 3×3 Grid-quorum Game

Similar to the 2×2 grid-quorum game, the 3×3 game has multiple NEs. In particular, for each fixed s_T, the game has four NEs. The utility function of this game is given by (10). For a given s_T, any pair of R $(s_{R,r}, s_{R,c})$ and J $(s_{J,r}, s_{J,c})$ strategies constitutes a NE if and only if $s_{J,r} = s_{T,r} \neq s_{R,r}$ and $s_{J,c} = s_{T,c} \neq s_{R,c}$. All NEs result in $u_T = -2$.

C. The $\sqrt{m} \times \sqrt{m}$ Grid-quorum Game

Theorem 2: For any $s_T = (s_{T,r}, s_{T,c})$, the $\sqrt{m} \times \sqrt{m}$ R/J grid-quorum game has at least $(\sqrt{m}-1)^2$ NEs, all of them result in $u_T = -2$. These NEs are given by:

$$
s_{J,r} = s_{T,r}, \quad s_{J,c} = s_{T,c} \tag{11}
$$

$$
s_{R,r} \neq s_{T,r}, \quad s_{R,c} \neq s_{T,c}. \tag{12}
$$

Proof: Assume that $s_{J,r} = s_{T,r}$ and $s_{J,c} = s_{T,c}$. Then, we want to show that if R deviates from the strategy given by (12), its utility u_R will be ≤ -2. R deviates from the strategy in (12) if it follows one of the following strategies:

1. $s_{T,r} = s_{R,r} = s_{J,r}$ and $s_{T,c} = s_{R,c} = s_{J,c}$
2. $s_{T,r} = s_{R,r} = s_{J,r}$ and $s_{T,c} = s_{J,c} \neq s_{R,c}$
3. $s_{T,r} = s_{J,r} \neq s_{R,r}$ and $s_{T,c} = s_{R,c} = s_{J,c}$.

From (7), in case 1, $u_R = 1 - 2\sqrt{m} < -2$. In cases 2 and 3, $u_R = -\sqrt{m} \leq -2$.

Now, assume that $s_{R,r} \neq s_{T,r}$ and $s_{R,c} \neq s_{T,c}$. Then, we want to show that if J deviates from the strategy given by (11), the resulted utility u_J will be ≤ 2. J deviates from the strategy in (11) if it follows one of the following strategies:

1. $s_{R,r} = s_{J,r} \neq s_{T,r}$ and $s_{R,c} = s_{J,c} \neq s_{T,c}$
2. $s_{R,r} \neq s_{T,r} \neq s_{J,r}$ and $s_{R,c} \neq s_{T,c} \neq s_{J,c}$
3. $s_{R,r} \neq s_{T,r} = s_{J,r}$ and $s_{R,c} = s_{J,c} \neq s_{T,c}$
4. $s_{R,r} \neq s_{T,r} = s_{J,r}$ and $s_{R,c} \neq s_{T,c} \neq s_{J,c}$
5. $s_{R,r} = s_{J,r} \neq s_{T,r}$ and $s_{R,c} \neq s_{T,c} = s_{J,c}$
6. $s_{R,r} = s_{J,r} \neq s_{T,r}$ and $s_{R,c} \neq s_{T,c} \neq s_{J,c}$
7. $s_{R,r} \neq s_{T,r} \neq s_{J,r}$ and $s_{R,c} \neq s_{T,c} = s_{J,c}$
8. $s_{R,r} \neq s_{T,r} \neq s_{J,r}$ and $s_{R,c} = s_{J,c} \neq s_{T,c}$.

From (7), u_J equals 2 for case 1, -2 for case 2, and 0 for cases 3 to 8. Hence, the pure strategies in (12) are pure NE strategies. ■

Proposition 1: The $(\sqrt{m}-1)^2$ NEs in Theorem 2 are the only NEs for the $\sqrt{m} \times \sqrt{m}$ grid-quorum game, when $m \geq 9$. When $m = 4$, the 2×2 game has additional NEs, given by:

$$
s_{T,r} = s_{R,r} = s_{J,r} \quad \text{and} \quad s_{J,c} = s_{T,c} \neq s_{R,c} \tag{13}
$$

$$
s_{J,r} = s_{T,r} \neq s_{R,r} \quad \text{and} \quad s_{T,c} = s_{R,c} = s_{J,c}. \tag{14}
$$

Proof: For the first part of the proposition, consider all strategies other than the NE strategies given by Theorem 2. We have eighteen strategies in (7), in addition to the strategies that result in $u_T = 0$, which are the following six strategies:

1. $s_{R,r} \neq s_{T,r} = s_{J,r}$ and $s_{R,c} = s_{J,c} \neq s_{T,c}$
2. $s_{R,r} \neq s_{T,r} = s_{J,r}$ and $s_{R,c} \neq s_{T,c} \neq s_{J,c}$
3. $s_{R,r} = s_{J,r} \neq s_{T,r}$ and $s_{R,c} \neq s_{T,c} = s_{J,c}$
4. $s_{R,r} = s_{J,r} \neq s_{T,r}$ and $s_{R,c} \neq s_{T,c} \neq s_{J,c}$
5. $s_{R,r} \neq s_{T,r} \neq s_{J,r}$ and $s_{R,c} \neq s_{T,c} = s_{J,c}$
6. $s_{R,r} \neq s_{T,r} \neq s_{J,r}$ and $s_{R,c} = s_{J,c} \neq s_{T,c}$.

It can be shown that none of the $18 + 6 = 24$ strategies is a NE. For the second part of the proposition, following the same procedure in the proof of Theorem 2, it can be shown that the strategies in (13) and (14) are indeed NEs. ■

Remark 4: Note that even though there are multiple pure NE strategies for a given s_T, the NE strategy of the jammer is unique. Furthermore, the utilities of the players do not depend on the NE strategy played by the receiver.

VI. Two-player Synchronous Rendezvous Game with Unknown Rendezvous Channel

In this section, we study the synchronous rendezvous game when the rendezvous channel is unknown to the players. We formulate this game as a Bayesian game. A Bayesian game is

$$u_T(s_T,s_R,s_J)=\begin{cases}-5, & \text{if } s_T=s_R=s_J\\ -3, & \text{if } s_{T,r}=s_{R,r}=s_{J,r} \text{ and } s_{R,c}\neq s_{T,c}, \text{ or } s_{R,r}\neq s_{T,r} \text{ and } s_{T,c}=s_{R,c}=s_{J,c}\\ -2, & \text{if } s_{T,r}=s_{J,r}\neq s_{R,r} \text{ and } s_{T,c}=s_{J,c}\neq s_{R,c}, \text{ or } s_{R,r}=s_{J,r}\neq s_{T,r} \text{ and } s_{R,c}=s_{J,c}\neq s_{T,c}\\ -1, & \text{if } s_{T,r}=s_{R,r}=s_{J,r} \text{ and } s_{T,c}=s_{R,c}\neq s_{J,c}, \text{ or } s_{T,r}=s_{R,r}\neq s_{J,r} \text{ and } s_{T,c}=s_{R,c}=s_{J,c}\\ 1, & \text{if } s_{R,r}=s_{T,r}\neq s_{J,r} \text{ and } s_{R,c}\neq s_{T,c}, \text{ or } s_{R,c}=s_{T,c}\neq s_{J,c} \text{ and } s_{R,r}\neq s_{T,r}\\ & \text{or } s_{T,r}=s_{R,r}\neq s_{J,r} \text{ and } s_{T,c}=s_{R,c}\neq s_{J,c}\\ 2, & \text{if } s_{R,r}\neq s_{T,r}\neq s_{J,r} \text{ and } s_{R,c}\neq s_{T,c}\neq s_{J,c}\\ 0, & \text{otherwise.}\end{cases} \tag{10}$$

characterized by a set of players, a set of pure strategies for each player, a set of types for each player, a payoff for each player, and a joint probability distribution over the types of the players [11]. Let $h_T, h_R, h_J \in \mathcal{L}$ denote the channels selected by T, R, and J, respectively. R is considered to have two types: $h_R = h_T$ and $h_R \neq h_T$. Similarly, J is of two types: $h_J = h_T$ and $h_J \neq h_T$. In Bayesian games, a pure strategy of a player is a map prescribing an action for each type of that player. The utility of player $i \in \{T, R, J\}$ is given by:

$$u_i(s_T,s_R,s_J,h_T,h_R,h_J)=\begin{cases}u_i(s_T,s_R,s_J), & \text{if } h_T=h_R=h_J\\ u_i(s_T,s_R), & \text{if } h_T=h_R\neq h_J\\ 0, & \text{otherwise (i.e., } h_T\neq h_R)\end{cases} \tag{15}$$

where $u_T(s_T,s_R,s_J) = u_R(s_T,s_R,s_J) = -u_J(s_T,s_R,s_J)$ is given by (7), and $u_T(s_T,s_R) = u_R(s_T,s_R) = -u_J(s_T,s_R)$ is given by:

$$u_T(s_T,s_R)=\begin{cases}2\sqrt{m}-1, & \text{if } s_T=s_R\\ \sqrt{m}, & \text{if } s_{T,r}=s_{R,r} \text{ and } s_{T,c}\neq s_{R,c}\\ & \text{or } s_{T,r}\neq s_{R,r} \text{ and } s_{T,c}=s_{R,c}\\ 2, & \text{if } s_{T,r}\neq s_{R,r} \text{ and } s_{T,c}\neq s_{R,c}.\end{cases} \tag{16}$$

The expected utilities of R of types $h_R = h_T$ and $h_R \neq h_T$ are given by:

$$\begin{aligned}&\mathbb{E}[u_R(s_T,s_R,s_J,h_R=h_T)]\\ &= \Pr\{h_J=h_T|h_R=h_T\}\,u_R(s_T,s_R,s_J)\\ &\quad+\Pr\{h_J\neq h_T|h_R=h_T\}\,u_R(s_T,s_R)\end{aligned} \tag{17}$$

$$\mathbb{E}[u_R(s_T,s_R,s_J,h_R\neq h_T)]=0. \tag{18}$$

The expected utilities of J of types $h_J = h_T$ and $h_J \neq h_T$ are given by:

$$\begin{aligned}&\mathbb{E}[u_J(s_T,s_R,s_J,h_J=h_T)]\\ &= \Pr\{h_R=h_T|h_J=h_T\}\,u_J(s_T,s_R,s_J)\end{aligned} \tag{19}$$

$$\begin{aligned}&\mathbb{E}[u_J(s_T,s_R,s_J,h_J\neq h_T)]\\ &= \Pr\{h_R=h_T|h_J\neq h_T\}\,u_J(s_T,s_R).\end{aligned} \tag{20}$$

Next, we derive the Bayesian NE.

Theorem 3: Let $p \stackrel{\text{def}}{=} \Pr\{h_J = h_T | h_R = h_T\}$, then the Bayesian NE of the synchronous rendezvous game with unknown rendezvous channel is given by:

$$s_R\begin{cases}\begin{cases}=s_T, & \text{if } p<0.5\\ \neq s_T, & \text{if } p>0.5\\ \text{Does not matter}, & \text{if } p=0.5\end{cases}, & \text{if } h_R=h_T\\ \text{Does not matter}, & \text{if } h_R\neq h_T\end{cases} \tag{21}$$

$$s_J\begin{cases}=s_T, & \text{if } h_J=h_T\\ \text{Does not matter}, & \text{if } h_J\neq h_T\end{cases} \tag{22}$$

where $s_R \neq s_T$ means that $s_{R,r} \neq s_{T,r}$ and $s_{R,c} \neq s_{T,c}$.

Proof: First, note that $s_J = s_T$ is a dominant jammer strategy, i.e.,

$$\begin{aligned}u_J(s_T,s_R,s_J=s_T,h_T,h_R,h_J) &\geq u_J(s_T,s_R,s'_J,h_T,h_R,h_J)\\ \forall s'_J\in S, s'_J\neq s_J, &\forall h_T,h_R,h_J\in\mathcal{L}.\end{aligned}$$

Now, if the jammer follows the strategy given by (22), then $\mathbb{E}[u_R(s_T,s_R,s_J,h_R = h_T)] = (1-2p)u_T(s_T,s_R)$. From (16), we have $2 \leq u_T(s_T,s_R) \leq 2\sqrt{m}-1$. Therefore, if $p < 0.5$, the maximum expected utility of the receiver is $(1-2p)(2\sqrt{m}-1)$ which occurs when $s_R = s_T$. If $p > 0.5$, the maximum expected utility of the receiver is $2(1-2p)$ which occurs when $s_R \neq s_T$. If $p = 0.5$ or $h_R \neq h_T$, then the expected utility of the receiver is 0 irrespective of s_R. Figure 5 depicts the $\mathbb{E}[u_R]$ at NE vs. p when $h_R = h_T$. ■

VII. Asynchronous Rendezvous

In the previous sections, we studied the rendezvous problem assuming all players are synchronized. However, it is difficult to ensure that all players are synchronized (i.e., start hopping at the same time). This is the motivation behind using the grid quorum system, which enjoys the rotation closure property.

Because the two-player R/J game is played assuming a fixed s_T, we consider the starting time of T as the reference point for R and J. Let θ_R denote the drift between R and T clocks, and θ_J denote the drift between J and T clocks. Without loss of generality, θ_R and θ_J are assumed to be $\leq m$. θ_R and θ_J are continuous variables, but they will be approximated as discrete integer variables, as explained next. We assume that one-half of a slot is enough to convey one

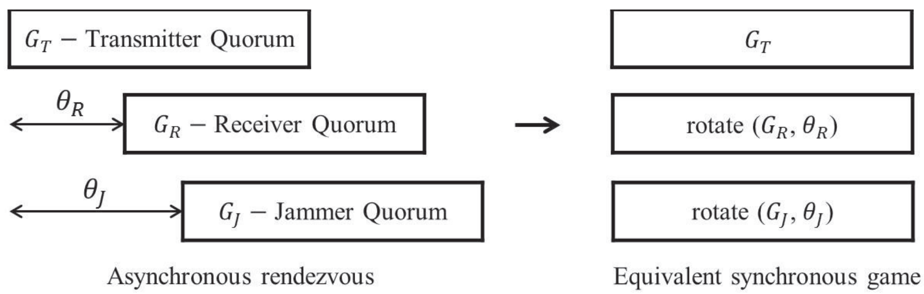


Fig. 6: The equivalent synchronous game between R and J for a given θ_R and θ_J.

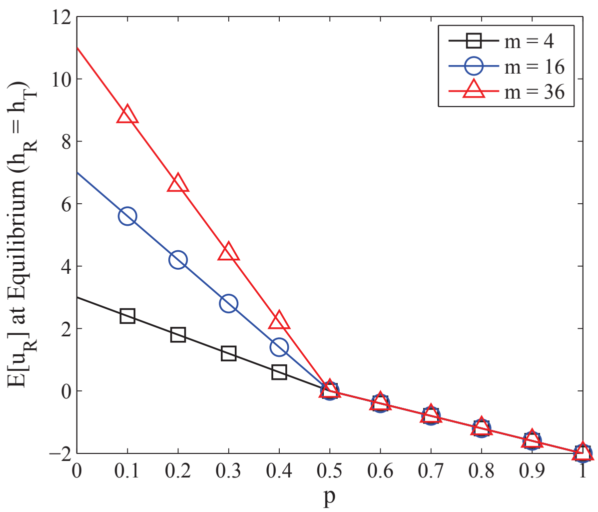


Fig. 5: Expected utility of the receiver at NE when $h_R = h_T$ as a function of p.

message between T and R, and that each rendezvous slot is used to convey only one message. Therefore, if T and R rendezvoused on less than half the duration of a give slot (i.e., $< T/2$ seconds), then this is the same as if they did not rendezvous on that slot. Similarly, if they met on a slot for more than half of its duration, then this is the same as if they met on the whole slot. Hence, if the fractional part of θ_R (similarly, θ_J) is < 0.5, it is discarded. If it is > 0.5, it is discarded after incrementing the integer part by one. So, θ_R and θ_J are considered as discrete variables taking values $0, 1, 2, \ldots, m$.

We will modify the strategy space of R and J in the asynchronous rendezvous case as follows. The strategy of the player (R or J) consists of a column, and a sequence of $\sqrt{m}$ consecutive elements that do not necessarily form a row. Let $s_R^{(a)}$ and $s_J^{(a)}$ denote R and J strategies, respectively. Then, $s_R^{(a)} \stackrel{\text{def}}{=} (s_{R,r}^{(a)}, s_{R,c}^{(a)})$ and $s_J^{(a)} \stackrel{\text{def}}{=} (s_{J,r}^{(a)}, s_{J,c}^{(a)})$. $s_{R,c}^{(a)}, s_{J,c}^{(a)} \in S_c^{(a)} = \{1, 2, \ldots, \sqrt{m}\}$ and $s_{R,r}^{(a)}, s_{J,r}^{(a)} \in S_r^{(a)}$, which is defined as follows:

$$\begin{aligned} S_r^{(a)} = \Big\{ & \left(n_1, \ldots, n_{\sqrt{m}}\right) : \\ & n_i \in \{0, 1, \ldots, m-1\}, \forall i \in \{1, \ldots, \sqrt{m}\} \\ & n_i - n_{i-1} = 1, \forall i \in \{2, \ldots, \sqrt{m}\} \Big\}. \end{aligned} \tag{23}$$

Let us denote $u_i(s_T, s_R, s_J)$, $i \in \{T, R, J\}$ by $u_i(G_T, G_R, G_J)$ when the selected quorums by T, R, and J are G_T, G_R, and G_J, respectively. Then, for any values of θ_R and θ_J, the asynchronous rendezvous problem is transformed into a synchronous rendezvous game as shown in Figure 6. From Figure 6, we have:

$$\begin{aligned} & u_i(s_T, s_R, s_J, \theta_R, \theta_J) = u_i(G_T, G_R, G_J, \theta_R, \theta_J) \\ & \quad = u_i(G_T, rotate(G_R, \theta_R), rotate(G_J, \theta_J)), \forall i \in \{T, R, J\}. \end{aligned} \tag{24}$$

Therefore, for a given $\theta_i, i \in \{R, J\}$, player i derives its NE strategy for the synchronous game (represented by $s_{i,r}$ and $s_{i,c}$), as given by Theorem 2. Then, from Figure 6, the asynchronous strategy represented by $s_{i,r}^{(a)} = (n_1, \ldots, n_{\sqrt{m}})$ and $s_{i,c}^{(a)}$ is computed as follows:

$$n_j = \sqrt{m} \times (s_{i,r} - 1) + (j - 1) - \theta_i, j = 1, \ldots, \sqrt{m} \tag{25}$$

$$s_{i,c}^{(a)} = \begin{cases} s_{i,c} - \theta_i, & \text{if } s_{i,c} > \theta_i \\ \sqrt{m} + (s_{i,c} - \theta_i), & \text{if } s_{i,c} \leq \theta_i. \end{cases} \tag{26}$$

Remark 5: R and J do not know their relative clock drifts with respect to the clock of T. Moreover, they do not know s_T. Therefore, each player does the following: (i) uniformly randomize between the different NE strategies that correspond to the different relative clock drifts of the player, and (ii) uniformly randomize between the different NE strategies that correspond to the different T strategies.

Remark 6: As stated before, we assume that θ_R and θ_J are at most one frame length. Accordingly, R and J are required to keep using their selected quorums (strategies) for only two successive frames, after which they can select another quorum. This condition is required to ensure the overlapping with the quorum of T, according to the rotation closure property.

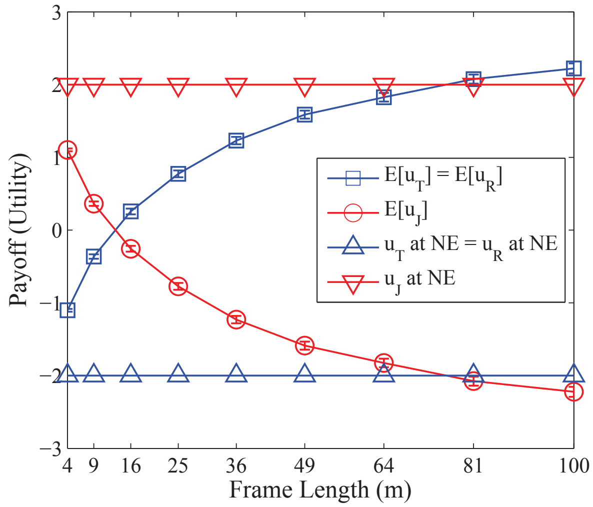


Fig. 7: Effect of m on the expected utilities of T, R, and J.

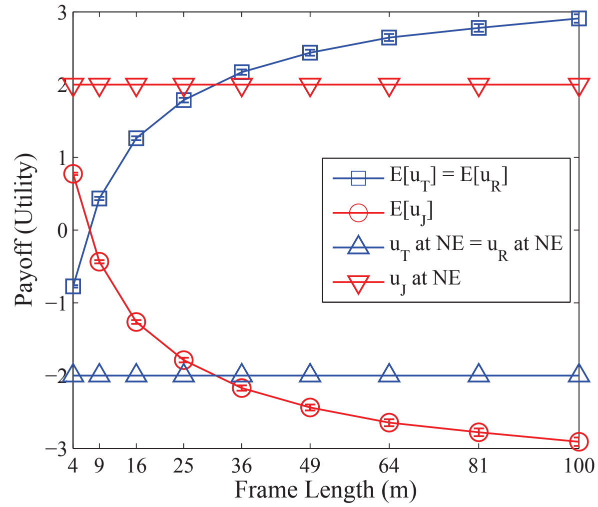


Fig. 8: Effect of m on the expected utilities of T, R, and J (R and J have a common guess about s_T).

VIII. Numerical Results

In this section, we study our formulated games numerically under different values of the system parameters. We implement our games in MATLAB. The 95% confidence intervals are indicated in the figures.

A. Synchronous Rendezvous

In this section, we simulate the synchronous rendezvous game between R and J, assuming a uniformly random strategy for T. Figure 7 plots the expected utilities of R and J vs. m. It also shows the utilities of R and J at the NE.

Observation 1: *R benefits from being, along with J, unaware of s_T. Furthermore, the benefits of R increase with m.*

As shown in Figure 7, the utility of R when s_T is unknown is always better than his NE utility when s_T is known. On the other hand, being unaware of s_T always harms J. As m increases, the randomness about s_T increases (recall that the strategy space of T is of dimension m), and the expected utility of R increases while the expected utility of J decreases.

Observation 2: *The utility of R improves if both R and J have the same guess about s_T.*

Figure 8 depicts the expected utilities of R and J when both have a common guess about s_T. By comparing the expected utility of R in Figure 7 with that in Figure 8, it is clear that the utility of R improves if both R and J have the same guess about s_T.

B. Asynchronous Rendezvous

In this section, we consider the asynchronous rendezvous case.

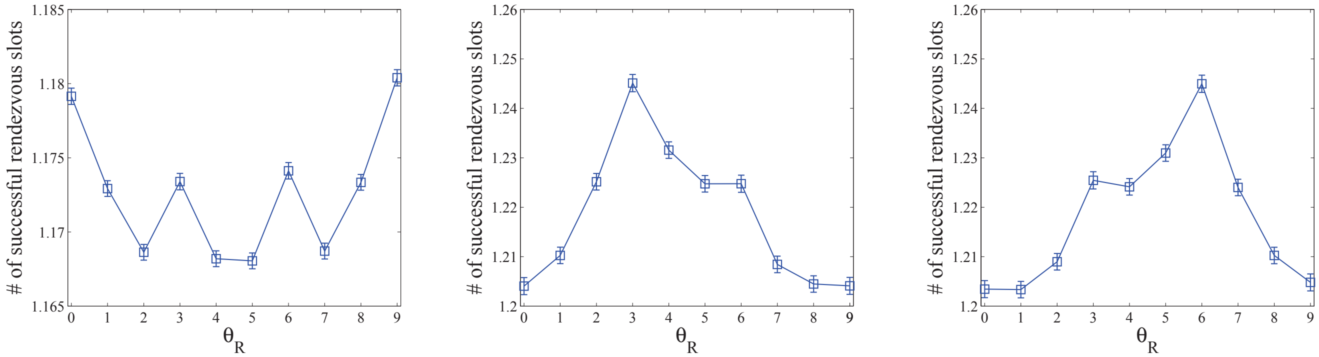
Observation 3: *The number of successful rendezvous slots is maximized when $\theta_R = \theta_J$.*

Figure 9 plots the number of successful rendezvous slots vs. θ_R for different values of θ_J. As shown in the figure, the number of successful rendezvous slots is maximized when $\theta_R = \theta_J$ (i.e., when R and J are synchronized). Note form Figure 9 also that the number of successful rendezvous slots is locally maximized at $\theta_R = 0, \sqrt{m}, 2\sqrt{m}, \ldots, m$, i.e., when T is ahead of R by an integer number of full rows (each row in the grid quorum system consists of $\sqrt{m}$ slots). When a quorum is cyclically rotated by $i\sqrt{m}$ slots for an integer i, the resulted quorum has the same column as the initial one but a different row. Because of this, the number of rendezvous slots is maximized when θ_R is an integer multiple of $\sqrt{m}$.

Figure 10 shows the number of successful rendezvous slots vs. θ_R for different values of θ_J, when R and J have a common guess about s_T. Again, the figure shows that the utility of R improves if both R and J have the same guess about s_T.

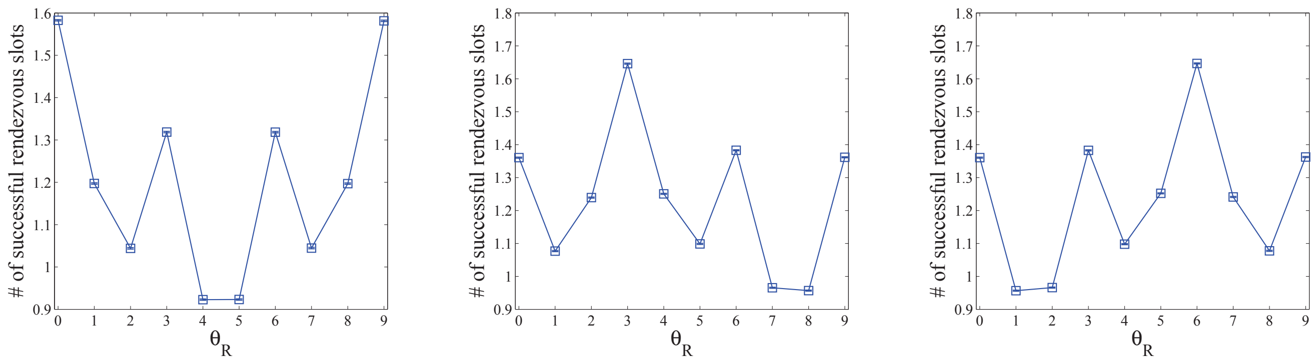
IX. Future Research: Nested Grid-Quorum Rendezvous Game

In this section, we provide a direction for extending the game-theoretic framework developed in this paper to the case when SUs use multiple rendezvous channels per frame (i.e., when the nesting degree is greater than one). This case is much more challenging than the non-nested case (i.e., the case when the nesting degree is equal to one) due to the following reason. In the non-nested case, all players (T, R, and J) play on a common grid quorum system, i.e., all players have the same strategy space. In the nested case, according to the NGQFH algorithm in Section III-B, the players will play $\sqrt{m} - 1$ games (when the frame length is m, and hence the nesting degree is $\sqrt{m} - 1$). The first game is played to select the outer-most quorum (a $\sqrt{m} \times \sqrt{m}$ quorum), and all players have the same strategy space (which consists of m strategies, as in the non-nested case). After playing the first game and selecting the outer-most quorums, the second game is played to select the next outer-most quorum. Since the selected outer-most quorums by the players in the first game are, in general, different, the strategy spaces (determined by the resulted sub-grids after removing the selected outer-most quorums) of the



(a) $\theta_J = 0$ (b) $\theta_J = 3$ (c) $\theta_J = 6$

Fig. 9: Expected number of successful rendezvous slots vs. θ_R for different values of θ_J ($m = 9$).



(a) $\theta_J = 0$ (b) $\theta_J = 3$ (c) $\theta_J = 6$

Fig. 10: Expected number of successful rendezvous slots vs. θ_R for different values of θ_J. R and J have a common guess about s_T ($m = 9$).

players in the second game are, in general, different. The same applies to the subsequent games (third, fourth, etc.). To illustrate further, consider the example in Figure 3. In this example, $m = 9$, and hence the nesting degree is $\sqrt{9} - 1 = 2$. Therefore, the players will play two games. The first game will be played on the common 3×3 grid, and the second game will be played on the two remaining 2×2 grids after removing the selected 3×3 quorums in the first game. So, the strategy spaces of the players in the second game are, in general, different.

To study the rendezvous game for the nested case, we propose formulating it as a matrix game with games as components [10]. In such type of matrix games, the outcome of a particular choice of pure strategies of the players may be that the players need to play another game. In our nested rendezvous game, the outcome of any choice of pure strategies (i.e., outer-most quorums in our case) of the players will consist of two parts. One part is obtained directly based on the selected pure strategies, and the other part depends on the subsequent games that will be played. In particular, our nested rendezvous game can be formulated as a multi-stage matrix game with games as components as follows. Consider, for example, the case of $m = 16$, hence the nesting degree is $\sqrt{16} - 1 = 3$. Let us consider the two-player R/J game. To simplify the notation, let us refer to each quorum by one index. We will refer to the quorum that consists of row i and column j by the index $(i-1)\sqrt{m} + j$. Let U_{R_4} be the receiver utility matrix of the game. Then, U_{R_4} can be written as follows.

$$U_{R_4} = \begin{pmatrix} a_{1,1} + U_{R_3}^{(1,1)} & \dots & a_{1,16} + U_{R_3}^{(1,16)} \\ a_{2,1} + U_{R_3}^{(2,1)} & \dots & a_{2,16} + U_{R_3}^{(2,16)} \\ \vdots & \ddots & \vdots \\ a_{16,1} + U_{R_3}^{(16,1)} & \dots & a_{16,16} + U_{R_3}^{(16,16)} \end{pmatrix} \quad (27)$$

where the (i, j) element in (27) consists of two parts: $a_{i,j}$ and $U_{R_3}^{(i,j)}$. $a_{i,j}$ is the utility (i.e., the number of successful rendezvous slots $-$ the number of jammed rendezvous slots) obtained from the outer-most quorums i and j, and $U_{R_3}^{(i,j)}$ is the expected utility of the subsequent game that will be played on the two 3×3 grids resulted after removing the outer-most quorum i from R's quorum system and outer-most quorum j from J's quorum system. However, $U_{R_3}^{(i,j)}$ is in turn a matrix game with games as components, which can be written as follows:

$$U_{R_3}^{(1,1)} = \begin{pmatrix} b_{1,1} + U_{R_2}^{(1,1)} & \dots & b_{1,9} + U_{R_2}^{(1,9)} \\ b_{2,1} + U_{R_2}^{(2,1)} & \dots & b_{2,9} + U_{R_2}^{(2,9)} \\ \vdots & \ddots & \vdots \\ b_{9,1} + U_{R_2}^{(9,1)} & \dots & b_{9,9} + U_{R_2}^{(9,9)} \end{pmatrix}. \quad (28)$$

Again, $b_{i,j}$ is the utility obtained from the 3×3 quorums i and j, and $U_{R_2}^{(i,j)}$ is the expected utility of the subsequent game that will be played on the two 2×2 grids resulted after removing the outer-most quorum i from R's quorum system and outer-most quorum j from J's quorum system.

Detailed analysis of the nested rendezvous game is left for future research.

X. Conclusions

In this paper, we studied the rendezvous problem in the presence of an insider attack using a game-theoretic framework. We considered the synchronous and asynchronous cases. Moreover, we investigated the cases when the rendezvous channel is known as well as when it is unknown. Our numerical results revealed that uncertainty about the transmitter's strategy improves the anti-jamming rendezvous performance. Moreover, the rendezvous performance improves if the receiver and jammer are synchronized, and also improves if the receiver and jammer have a common guess about the transmitter's strategy. Finally, we provided a direction for extending the developed game-theoretic framework to the case when SUs use multiple rendezvous channels per frame (i.e., when the nesting degree is greater than one).

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