Distributed Bargaining Mechanisms for MIMO Dynamic Spectrum Access Systems

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***Abstract*—Dynamic spectrum access (DSA) and MIMO communications are among the most promising solutions to address the ever increasing wireless traffic demand. An integration that successfully embraces the two is far from trivial due to the dynamics of spectrum opportunities as well as the requirement to jointly optimize both spectrum allocation and spatial/antenna pattern in a *distributed fashion*. Regardless of spectrum dynamics and heterogeneity, existing literature on channel/power allocation in MIMO DSA systems is only applicable to centralized cases. Our objective here is to design distributed algorithms that jointly allocate opportunistic channels to various links and to simultaneously optimize the MIMO precoding matrices so as to achieve fairness or maximize network throughput. For self-interested DSA links, our distributed algorithm allows links to negotiate channel allocation based on Nash bargaining (NB) and configure the precoding matrices so that links' rate demands are guaranteed while the surplus resources (after meeting minimum rate demands) are fairly allocated. Next, we consider a network throughput maximization formulation (NET-MAX). Both the NB-based and NET-MAX problems are combinatorial with mixed variables. To tackle them, we first transform the original problems by incorporating the concept of timesharing. Using dual decomposition, we develop optimal distributed algorithms for timesharing case, which shed light on how to derive a distributed algorithm for the original problems. Our work fills a gap in the literature of channel allocation where a central controller is not available.**

***Index Terms*—Nash bargaining, dual decomposition, distributed algorithm, throughput maximization, cognitive radio, MIMO precoding, fairness, rate demands.**

I. Introduction

Mobile data traffic has grown explosively in recent years and is estimated to increase more than one thousand-fold in the next 10 years [2]. Dynamic spectrum access (DSA) and multi-input multi-output (MIMO) communications are among the most promising solutions to address this ever increasing wireless demand. In a DSA system, secondary users (SUs) communicate opportunistically on temporarily idle or under-utilized portions of the licensed spectrum. MIMO systems boost spectral efficiency by allowing a multi-antenna node to simultaneously transmit multiple data streams (i.e., *spatial multiplexing*). Recent standards (e.g., 4G Advanced-LTE, IEEE 802.16e, IEEE 802.11ac) adopt MIMO communications as a core feature. The FCC has opened up TV white bands for opportunistic, secondary use [3]. A timely issue is to integrate both technologies into a single system.

MIMO transmitters can realize spatial multiplexing by employing the *precoding technique*, in which a vector of information symbols (one symbol per data stream) is pre-multiplied by a matrix, called a *precoder*, before being transmitted over an antenna array [4]. By adjusting the amplitude and tuning the phase of each complex element in the precoding matrix, one can control the powers allocated to various data streams and the antenna's radiation directions. On the other hand, through channel bonding/aggregation, a DSA user with cognitive radio (CR) capabilities can simultaneously transmit over several channels (in the frequency domain), obtained either from spectrum databases/brokers [3] or through sensing. Given a pool of temporarily idle channels and a number of MIMO-capable secondary devices, one critical issue is how to jointly assign channels to various links and simultaneously optimize the MIMO precoders so as to maximize a system objective (e.g., network throughput or proportional fairness, etc.) while meeting links' rate demands.

The problem of joint channel/power allocation for a set of links, even for a network of single-antenna devices and without the need to protect primary users (PUs), is known to be NP-hard [5]. In an opportunistic MIMO-capable DSA network with spatial multiplexing, the problem is even more challenging. First, channel assignment for various links must be done in a dynamic and adaptive manner to harvest link/user and frequency diversities. Second, this channel assignment has to take into account both MIMO antenna radiation directions and the powers allocated to various data streams over both the space/antenna and frequency dimensions (see Fig. 1(c)). Mathematically, we face a combinatorial optimization problem with a large number of mixed variables, which increases quadratically with the antenna array size. Third, the spectrum opportunities are heterogeneous for different secondary links, i.e., the set of available channels changes from one link to another (see Fig. 1(d)). Fig. 1 illustrates an example where several TV whitespaces are allocated to 3 links with nonidentical sets of available channels ($S1$,

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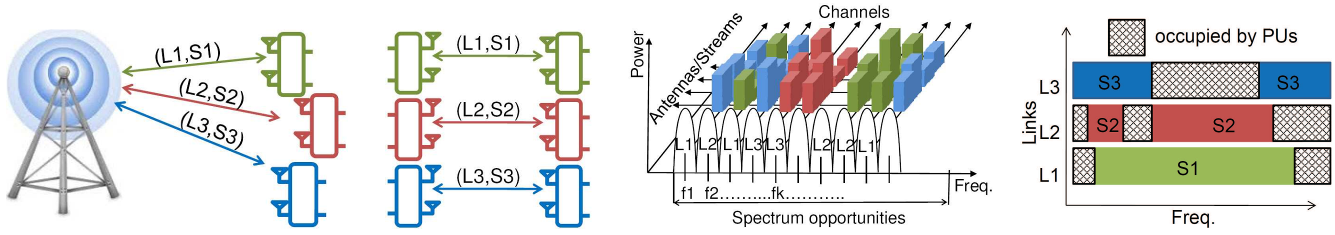

(a) (b) (c) (d)

Fig. 1. (a) MIMO-capable DSA network with an access point/database or spectrum broker, (b) ad hoc MIMO-capable DSA network, (c) channel assignment and power allocation over both antenna/space and frequency dimensions, (d) heterogeneity of spectrum opportunities (S_i refers to the idle spectrum as seen by link L_i).

$S2$, and $S3$). Forth, considering a context in which DSA users are administered by different entities that are distributed in nature, existing centralized channel allocation algorithms (e.g., [5] [6] [7]) are inapplicable.

This article designs *distributed algorithms* that allow MIMO-capable SUs, referred to as cognitive MIMO (CMIMO) nodes, to cooperate/bargain to determine their assigned channels and optimally design their Tx/Rx precoders to either achieve fairness or maximize network throughput under a heterogeneous spectrum scenario. For the fairness objective, we propose a bargaining framework, referred to as BF-CMIMO in which the surplus/remaining resources after meeting links' rate demands are shared in a proportionally fair manner. Bargaining-based resource allocation, e.g., Nash bargaining (NB) [8], is a type of cooperative games that often yields superior performance to noncooperative ones [9] [10] [11] [12]. However, such an approach is often centralized (requires the assistance of an *arbitrator* to manage the bargaining process). These works also do not support exclusive channel occupancy. The challenge that hinders a fully distributed bargaining algorithm with exclusive channel occupancy is the combinatorial complexity of the joint power/channel allocation problem, which includes both integer and real variables. Even by relaxing the integer variables, the problem is still non-convex. The network throughput maximization problem (NET-MAX) also faces similar challenges.

Methodologically, to deal with the joint power and channel allocation problem, seminal works (e.g., [5] [6] [7]) relied on the zero-duality gap (under the assumption of a theoretically infinite number of channels) to solve the problem in the dual domain. Unfortunately, this technique is only applicable to the centralized scenario (e.g., downlink case) but not distributed cases (e.g., cellular uplinks or ad hoc networks). Specifically, for the NET-MAX problem, [5] [6] [7] decomposed the problem in its dual domain w.r.t. carriers/channels. A central computer is then used to solve various per-carrier subproblems and find by trial and error the most suitable link for a given channel. In a DSA setting, CR nodes may belong to different administrative entities that are distributed in nature. Hence, the existence of a central controller is not guaranteed. Digging deeper, given the aforementioned zero-duality gap, one would want to decompose the problem w.r.t. links in its dual domain to obtain a distributed algorithm. However, the resulting subproblems in the dual domain still face the same combinatorial selection of channels as in the primal problem. Moreover, for the BF-CMIMO framework, the decomposition w.r.t. carriers/channels is not even possible due to the product-structure of its objective function (e.g., [8] [9] [10] [11] [13] [12]).

To overcome the aforementioned challenges in this paper, we transform the initial BF-CMIMO and NET-MAX formulations into their equivalent ones, whose relaxed versions are convex. This relaxed problems serve two purposes. First, the relaxed variable can be interpreted as a "timesharing factor" that represents the fraction of time a channel is allocated to a given link. Hence, this relaxed version is of practical interest when synchronization among links is possible. Using dual decomposition [14], we develop distributed algorithms for the relaxed timesharing problems of both BF-CMIMO and NET-MAX and prove their convergence to the globally (centralized) optimal solutions. Second, the distributed bargaining algorithms under timesharing allow us to gauge the preferences of different CMIMO links for a give channel (quantified by a "payoff" vector). Using these preferences, we develop a heuristic distributed algorithms for the original BF-CMIMO and NET-MAX formulations.

Simulations indicate that the heuristic distributed algorithms for BF-CMIMO and NET-MAX attain 95% performance of their optimal solutions (found via exhaustive search), and are within 87% and 78% of the upper bound on the performance of their optimal solutions (obtained under the timesharing assumption). When the number of channels is large, the distributed NET-MAX algorithm achieves almost the same performance as the centralized (optimal) one, which is based on the zero-duality gap (e.g., [6]). Table I compares existing algorithms that exploit the zero-duality gap and our proposed algorithms. Our proposed schemes and formulations supplement the MIMO spectrum sharing literature e.g., [15] [16] [17] [18] [19], where a given channel is simultaneously shared by several links. These works assume homogeneous spectrum, use a noncooperative game model and do not consider rate demands and fairness. In contrast, we consider an exclusive channel allocation policy (amenable to multi-carrier systems like OFDMA), modeled as a cooperative game with heterogeneous spectrum opportunities.

TABLE I
COMPARISON BETWEEN ZERO-DUALITY BASED APPROACH AND OUR PROPOSED APPROACH.

Methods	BF-CMIMO		NET-MAX	
	Distributed	Centralized	Distributed	Centralized
Zero-duality Gap (e.g., [5] [6] [7])	Inapplicable	Inapplicable	Inapplicable	Applicable (only for a large # of channels)
Proposed approach	Applicable	Applicable	Applicable	Applicable

In Section II, we present the network model and problem formulation. Centralized and distributed algorithms for BF-CMIMO are presented in Section III. Section IV addresses the NET-MAX problem. Numerical results are discussed in Section V, followed by concluding remarks in Section VI.

Throughout the paper, we use $(.)^H$ for the Hermitian transpose, tr$(.)$ for the trace of a matrix, $|.|$ for the determinant, and $\text{eig}_{\max}(.)$ for the maximum eigenvalue of a matrix. Matrices and vectors are indicated in boldface.

II. PROBLEM FORMULATION

A. Network Model

We consider a CMIMO network of N links. Each node is equipped with M antennas (our analysis is also applicable when nodes have different numbers of antennas). The set of channels available for link i (i.e., temporarily not occupied by PUs) is denoted by $\mathbf{S}_i$. In general, $\mathbf{S}_i \neq \mathbf{S}_j$ for two links i and j, although due to their proximity the two links are likely to share many channels. The network's opportunistic spectrum is the union of available-channel sets from all links $\Psi_K \stackrel{\text{def}}{=} \bigcup_{i=1}^{N} \mathbf{S}_i$, consisting of K orthogonal (not necessarily contiguous) channels with central frequencies f_1, f_2, ..., f_K (for simplicity, we use the same notation f_k to refer to the kth channel). Let $\Phi_N \stackrel{\text{def}}{=} \{1, 2, \ldots, N\}$ be the set of CMIMO links. At a given time, each link i may simultaneously communicate over a set of channels, denoted by $\mathbf{A}_i$.

Our setup supports an exclusive channel occupancy scheme, i.e., each channel is exclusively allocated to no more than one SU link or $\mathbf{A}_i \cap \mathbf{A}_j = \emptyset, \forall i \neq j$. First, if a channel is simultaneously shared by more than one link, the accumulated interference from multiple uncoordinated SUs cannot be controlled (even if we impose a power mask constraint, as specified by FCC in [3]). Consequently, excessive interference from SUs can harm PU transmissions. Second, in contrast to existing works in the literature (e.g., [15] [16] [17] [20] [18]) where all SUs share the same pool of channels (homogeneous spectrum sharing), we consider a heterogeneous spectrum setting (i.e., the set of spectrum opportunities/channels varies from one SU to another). In this case, allocating a channel to more than one link may not always be possible. Finally, the exclusive channel occupancy model is more amenable to protocol implementation via the widely used multi-carrier systems like OFDMA or Non-Contiguous OFDM in cognitive radio networks.

Let $\mathbf{A} = [a_{i,k}]$ be an $N \times K$ matrix that represents the channel assignment; $a_{i,k} = 1$ if channel f_k is allocated to link i, otherwise $a_{i,k} = 0$.

On a given allocated channel, a transmitting node can send up to M independent data streams using its M antennas. Formally, for channel f_k, let $\mathbf{x}_{i,k}$ with $E[\mathbf{x}_{i,k}\mathbf{x}_{i,k}^H] = \mathbf{I}$ be a column vector of M information symbols, sent from transmitter i to its receiver. Each element of $\mathbf{x}_{i,k}$ is from one data stream. Let $\tilde{\mathbf{T}}_{i,k} \in \mathbb{C}^{M\times M}$ denote the precoding matrix of node i on channel f_k. Then, the transmit vector is $\tilde{\mathbf{T}}_{i,k}\mathbf{x}_{i,k}$ and the received signal vector $\mathbf{y}_{i,k}$ is given by:

$$\mathbf{y}_{i,k} = \mathbf{H}_i^{(k)}\tilde{\mathbf{T}}_{i,k}\mathbf{x}_{i,k} + \mathbf{n}_k \tag{1}$$

where $\mathbf{H}_i^{(k)}$ is an $M \times M$ channel gain matrix for channel f_k on link i and $\mathbf{n}_k \in \mathbb{C}^M$ is an $M \times 1$ complex Gaussian noise vector with identity covariance matrix $\mathbf{I}$, representing the floor noise plus normalized (and whitened) interference on channel k. Each element of $\mathbf{H}_i^{(k)}$ is the multiplication of a distance- and channel-dependent attenuation term, and a random term that reflects multi-path fading (assumed to be a complex Gaussian variable with zero mean and unit variance).

We assume flat fading and the channel state information (CSI) $\mathbf{H}_i^{(k)}$ is available at SU transmitters via conventional channel estimation methods for PUs (e.g., training sequences embedded into the handshaking packets of MAC protocols or blind channel estimation [21]). Robust bargaining models with imperfect or partial CSI are left for future work. The Shannon rate for link i on channel f_k is [4]:

$$R_{i,k} = \log|\mathbf{I} + \tilde{\mathbf{T}}_{i,k}^H\mathbf{H}_i^{(k)H}\mathbf{H}_i^{(k)}\tilde{\mathbf{T}}_{i,k}|. \tag{2}$$

Note that in the above, the information vector $\mathbf{x}_{i,k}$ and noise $\mathbf{n}_k$ are absorbed in equation (2) as we assume $E[\boldsymbol{x}_{i,k}\boldsymbol{x}_{i,k}^H] = \mathbf{I}$ [1] and the normalized noise-plus-interference (from primary users) vector $\mathbf{n}_k$ has an identity covariance matrix $\mathbf{I}$.

The total rate over all channels assigned to link i is:

$$R_i = \sum_{k\in\mathbf{S}_i} a_{i,k}R_{i,k}. \tag{3}$$

Each link i is subject to a rate demand c_i, i.e., we require that $R_i \geq c_i$. PUs are protected through database-authorized access and frequency-dependent power masks on secondary transmissions. Note that FCC specifications [3] impose power masks on opportunistic transmissions even over idle channels, if such channels are adjacent to PU-active channels (e.g., this power mask is 40 mW for bands adjacent to active TV

[1] The power allocation over M antennas/streams of a transmitter is realized through its precoding matrix, not information symbols

bands). Let $\mathbf{P}_{\text{mask}} \stackrel{\text{def}}{=} (P^{(1)}_{\text{mask}}, P^{(2)}_{\text{mask}}, \ldots, P^{(K)}_{\text{mask}})$ denote the vector of power masks over various channels. Let $P^{(i)}_{s,k}$ denote the allocated power on channel k and antenna s of link i, we require:

$$\sum_{s=1}^{M} P^{(i)}_{s,k} = \text{tr}(\tilde{\mathbf{T}}^H_{i,k}\tilde{\mathbf{T}}_{i,k}) \leq P^{(k)}_{\text{mask}}, \forall i \in \Phi_N \text{ and } \forall k \in \Psi_K. \tag{4}$$

To account for spectrum heterogeneity, we force link i not to transmit on channels that are not available for its use by imposing a link-dependent power-mask vector. For link i, $\mathbf{P}_{\text{mask},i} \stackrel{\text{def}}{=} (P^{(1)}_{\text{mask},i}, P^{(2)}_{\text{mask},i}, \ldots, P^{(K)}_{\text{mask},i})$, where $P^{(k)}_{\text{mask},i} = 0$ if $f_k \notin \mathbf{S}_i$, and $P^{(k)}_{\text{mask},i} = P^{(k)}_{\text{mask}}$ otherwise. Note that $\mathbf{P}_{\text{mask},i}$ differs from one link to another.

For link i, the total power allocated on all channels and all antennas should not exceed a limit $P_{\max}$ (without loss of generality, we assume the same power limit for all secondary users). Consequently,

$$\sum_{k\in\mathbf{S}_i}\sum_{s=1}^{M} P^{(i)}_{s,k} = \sum_{k\in\mathbf{S}_i} \text{tr}(\tilde{\mathbf{T}}^H_{i,k}\tilde{\mathbf{T}}_{i,k}) \leq P_{\max}. \tag{5}$$

B. Nash Bargaining Formulation

1) Overview of Bargaining Games: Bargaining is a special type of cooperative games where players negotiate/bargain their actions/strategies to reach an agreement that guarantees minimum payoffs (otherwise, players would act independently). The agreement is associated with a utility vector $\boldsymbol{u} \stackrel{\text{def}}{=} (u_1, \ldots, u_N)$, where u_i is the utility of player i, $i = 1, \ldots, N$. Let b_i and B_i denote the action and action space for player i, respectively ($b_i \in B_i$). The utility u_i is a function of the action vector $\boldsymbol{b} \stackrel{\text{def}}{=} (b_1, \ldots, b_N)$. The utility space U is the set of all possible payoff allocations $\boldsymbol{u}$ that result from all possible action vectors $\boldsymbol{b}$. It is also possible that no agreement is reached after bargaining, a situation referred to as a *disagreement point*. A disagreement point is associated with a minimum payoff vector $\boldsymbol{u}^0$.

In [8], Nash proposed axioms that define a Nash bargaining solution (NBS). An NBS guarantees all links' demands and is Pareto-optimal, meaning that there is no other solution that simultaneously leads to better payoffs for two or more players. Nash proved that if U is upper-bounded, closed, and convex, then there exists a unique NBS, which is obtained by solving the following problem:

$$\underset{\{\boldsymbol{b}\in B\}}{\text{maximize}} \prod_{i=1}^{N} (u_i - u_i^0). \tag{6}$$

Even if U is not convex, the NBS may still exist. Though a convex utility space makes the bargaining process more tractable, cases with nonconvex utility spaces are common, such as the one in this paper.

Under the Pareto-optimal NBS, players maximize the product of their *surplus* utilities, obtained by deducting their actual payoffs with their demands. Intuitively, the surplus utilities come from the fact that players are willing to cooperate to reach an agreement where their minimum demands are met and the resulting gain/surplus from cooperating is fairly allocated. Thus, NB has been shown to be a generalized version of the proportionally fair resource allocation mechanism in [13].

2) Bargaining Formulation for CMIMO Systems: To achieve fairness in sharing surplus resources (after meeting minimum rate demands) in a CMIMO network, we propose a variant bargaining framework of NBS[2], called BF-CMIMO. In BF-CMIMO, nodes are allowed to propose their rate demands. They then jointly allocate spectrum and optimize their precoders in a distributed manner. We map links to bargainers/players. The action of player i is $(\mathbf{A}_i, \tilde{\mathbf{T}}_i)$, where $\tilde{\mathbf{T}}_i \stackrel{\text{def}}{=} \{\tilde{\mathbf{T}}_{i,k}, k \in \mathbf{A}_i\}$ is the set of precoding matrices for the set of channels available to i. Player i's utility is its transmission rate R_i. We aim at finding a channel allocation matrix $\mathbf{A}$ and sets of precoders for all transmitters $(\tilde{\mathbf{T}}_i, \forall i \in \Phi_N)$ that solve the following problem:

BF-CMIMO Formulation:

$$\begin{aligned}
&\underset{\{\forall i\in\Phi_N, a_{i,k}, \tilde{\mathbf{T}}_{i,k}, \forall k\in\mathbf{S}_i\}}{\text{maximize}} \sum_{i\in\Phi_N} \log\Big(\sum_{k\in\mathbf{S}_i} a_{i,k}R_{i,k} - c_i\Big)\\
&\text{s.t.}\quad \text{C1: } \sum_{k\in\Psi_K} a_{i,k}R_{i,k} \geq c_i, \forall i \in \Phi_N\\
&\qquad\ \ \text{C2: } \text{tr}(\tilde{\mathbf{T}}^H_{i,k}\tilde{\mathbf{T}}_{i,k}) \leq P^{(k)}_{\text{mask},i}, \forall k \in \mathbf{S}_i, \forall i \in \Phi_N\\
&\qquad\ \ \text{C3: } \sum_{k\in\Psi_K} \text{tr}(\tilde{\mathbf{T}}^H_{i,k}\tilde{\mathbf{T}}_{i,k}) \leq P_{\max}, \forall i \in \Phi_N\\
&\qquad\ \ \text{C4: } \sum_{i\in\Phi_N} a_{i,k} \leq 1, \forall k \in \mathbf{S}_i\\
&\qquad\ \ \text{C5: } a_{i,k} = \{0, 1\}, \forall k \in \mathbf{S}_i, \forall i \in \Phi_N
\end{aligned} \tag{7}$$

where C1 guarantees the minimum rate requirement for all CR links, C2 ensures that PU reception is protected from CR transmissions, C3 is the maximum power constraint of link i, C4 and C5 convey the exclusive-channel occupancy policy.

Remark 1: The minimum rate requirement c_i depends on the link i's upper-layer application. Before joining the bargaining process, link i obtains an initial channel assignment $\mathbf{A}^0_i$ from its spectrum database that guarantees c_i. If c_i cannot be met even with all the channels available for the link in set $\mathbf{S}_i$, link i should either reduce its requested rate or wait for more spectrum opportunities to become available (to be added to $\mathbf{S}_i$) before bargaining.

Remark 2: The general scenario under consideration (Fig. 1(b) in the paper) differs fundamentally from conventional resource allocation scenarios. In conventional scenarios, resources are owned by a single entity, who is in charge of allocating these resources to her users. In our case, resources may be owned by different players/links and no one controls the whole system's resources (to apply traditional fairness schemes, e.g., max-min). Besides, the ad hoc nature of SUs requires a distributed bargaining mechanism among them. Resource allocation in our case is governed by a bargaining/negotiation among SUs.

For a special case (Fig. 1(a) in the paper) where a spectrum broker can govern the resource allocation process, conventional fairness schemes such as max-min can be applicable. However, unlike conventional resource allocation,

[2] NBS-based resource allocation generalizes proportionally fair resource allocation [13]. Resources are first allocated to meet players' minimum requirements, and the remaining resources are then proportionally allocated to all players.

SU links/users who have their own rate demands and resources do not necessarily share the interest in achieving a fair resource allocation, but rather they are interested in improving their own utilities/rates (i.e., SUs are rational). Hence, existing fairness schemes (e.g., max-min) in which all parties overlook their individual interests to achieve a common goal of fair resource allocation are not relevant to our setup. Regarding the comparison with conventional fairness approaches, it has been shown in [13] that NB (and thus its variant BF-CMIMO) is a generalized version of the proportionally fair resource allocation mechanism. It is well-known (e.g., [22] [23] [24]) that the max-min fairness approach can achieve better fairness [23] but lower total throughput than the proportionally fair approach [24][3] (a better balance between fairness and network throughput).

Problem (7) is combinatorial w.r.t. the binary variables $a_{i,k}$'s and the continuous variables in $\tilde{\mathbf{T}}_{i,k}$. Even a centralized solution would be computationally expensive, with a worst-case exponential complexity. Approximate solutions to binary programming problems can be obtained by relaxing the integer constraints (allowing $a_{i,k}$ to be a real number from 0 to 1), followed by sequential fixing. However, relaxing $a_{i,k}$ does not make (7) convex, as its objective function is not concave w.r.t. *both* the channel allocation indicator and the set of precoders $(a_{i,k}, \tilde{\mathbf{T}}_i)$ (although R_i is concave w.r.t. the set of precoders $\tilde{\mathbf{T}}_i$). Moreover, even if a centralized solution to (7) were to be found, it would still be impractical for distributed operation.

C. Network Throughput Maximization Formulation

Note that BF-CMIMO serves as a partially cooperative scheme in which links cooperate given that their minimum requirements are met *and* the throughput gain from cooperation is proportionally allocated [13]. For the network throughput maximization (NET-MAX) formulation, links can choose to cooperate in a tighter manner (by dropping the proportional fairness) to maximize the network throughput. This is done by replacing the objective function in (7) with the total network throughput:

NET-MAX Formulation:

$$\begin{aligned}&\underset{\{a_{i,k},\tilde{\mathbf{T}}_{i,k},\forall k\in \boldsymbol{S}_i,\forall i\in\Phi_N\}}{\text{maximize}} \sum_{i\in\Phi_N}\sum_{k\in\boldsymbol{S}_i} a_{i,k}R_{i,k} \\ &\text{s.t.} \quad \text{C1, C2, C3, C4, C5 in (7).}\end{aligned} \tag{8}$$

We emphasize that NET-MAX can be addressed in a centralized manner by exploiting its zero duality gap, revealed in [5] [6] [7]. However, this approach does not apply to distributed scenarios, as explained in Section IV.

Because each channel can be assigned to one link only, the best strategy for the Tx and Rx of a given MIMO link is to design their precoders to align M data streams to subchannels [4] obtained using the singular-value decomposition:

$$\mathbf{H}_{i,i}^{(k)} = \mathbf{U}_{i,k}\mathbf{G}_{i,k}\mathbf{T}_{i,k}^H \tag{9}$$

where $\mathbf{U}_{i,k}$ and $\mathbf{T}_{i,k}$ are unitary matrices, and $\mathbf{G}_{i,k}$ is a diagonal matrix formed from the singular values $g_{s,k}^{(i)}$, $s = 1,\ldots,M$, of the channel matrix $\mathbf{H}_{i,i}^{(k)}$. At the transmitter, we set $\tilde{\mathbf{T}}_{i,k}$ to $\mathbf{T}_{i,k}\mathbf{P}_k^{(i)\,1/2}$ [4], where $\mathbf{P}_k^{(i)}$ is a diagonal matrix whose sth diagonal element $P_{s,k}^{(i)}$ is the power allocated to stream s on channel f_k of link i. The achievable rate over channel f_k is $R_{i,k} = \sum_{s=1}^{M}\log(1+g_{s,k}^{(i)}P_{s,k}^{(i)})$.

We can rewrite (7) as follows:

$$\begin{aligned}&\underset{\{\forall i\in\Phi_N, a_{i,k}, P_{s,k}^{(i)}\}}{\text{maximize}} \sum_{i\in\Phi_N}\log\Big(\sum_{k\in\mathbf{S}_i}(a_{i,k}\sum_{s=1}^{M}\log(1+g_{s,k}^{(i)}P_{s,k}^{(i)}))-c_i\Big)\\
\text{s.t.}\quad &\text{C1'}: \sum_{k\in\Psi_K} a_{i,k}\sum_{s=1}^{M}\log(1+g_{s,k}^{(i)}P_{s,k}^{(i)}) \geq c_i, \forall i\in\Phi_N\\
&\text{C2'}: \sum_{s=1}^{M} P_{s,k}^{(i)} \leq P_{\text{mask},i}^{(k)}, \forall k\in\Psi_K, \forall i\in\Phi_N\\
&\text{C3'}: \sum_{k\in\Psi_K}\sum_{s=1}^{M} P_{s,k}^{(i)} \leq P_{\max}, \forall i\in\Phi_N\\
&\text{C4'}: \sum_{i\in\Phi_N} a_{i,k} \leq 1, \forall k\in\Psi_K\\
&\text{C5'}: a_{i,k} = \{0,1\}, \forall k\in\Psi_K, \forall i\in\Phi_N.\end{aligned} \tag{10}$$

Similarly, problem (8) becomes:

$$\begin{aligned}&\underset{\{a_{i,k}, P_{s,k}^{(i)}\}}{\text{maximize}} \sum_{i\in\Phi_N}\sum_{k\in\mathbf{S}_i} a_{i,k}\sum_{s=1}^{M}\log(1+g_{s,k}^{(i)}P_{s,k}^{(i)})\\ &\text{s.t.}\quad \text{C1', C2', C3', C4', C5' in (10)}\end{aligned} \tag{11}$$

III. Distributed Bargaining Algorithm

A. Timesharing Interpretation

Problem (10) is NP-hard [5]. Moreover, the decomposition approach w.r.t. channels in [5] [6] [7] is not possible due to the product structure of the objective function in (10). If we relax the binary constraint C5', its relaxed version is not convex as the objective function is not concave w.r.t. $(a_{i,k}, P_{s,k}^{(i)})$. To address (10) and provide a distributed algorithm, lets consider the following function:

$$f(a_{i,k}, P_{s,k}^{(i)}) \overset{\text{def}}{=} \begin{cases} a_{i,k}\sum_{s=1}^{M}\log(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}}) & \text{if } 0 < a_{i,k} \leq 1\\ 0 & \text{if } a_{i,k} = 0.\end{cases} \tag{12}$$

The idea of introducing function $f(a_{i,k}, P_{s,k}^{(i)})$ is inspired by [25]. The intuition behind the function can be interpreted as either frequency/channel sharing (e.g., [26]) or time sharing (as in [12] [25]). In both cases, $a_{i,k}\sum_{s=1}^{M}\log(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}})$ is the throughput of link i if link i is *exclusively* allocated a fraction $a_{i,k}$ (either in time or bandwidth) of channel k. When the channel is exclusively allocated to link i (either in time or frequency), there is no interference, hence the signal-to-noise ratio on each data stream (or interpreted on antenna s) is the product of the subchannel s gain $g_{s,k}^{(i)}$ (of link i on channel k) and transmit power divided by the noise-plus-interference power. Note that in our setup we normalize the noise-plus-interference power to be one. Under frequency/channel

[3] "Price of proportional fairness is substantially smaller than the price of max-min fairness", page 2 in [24].

sharing (e.g., [26], p. 359), the physical meaning of $\frac{P_{s,k}^{(i)}}{a_{i,k}}$ is the ratio of the transmit power $P_{s,k}^{(i)}$ and the noise-plus-interference power (one per-spectrum-unit times the shared bandwidth fraction $a_{i,k}$). Under time sharing (e.g., [12] [25]), the optimization variable $P_{s,k}^{(i)}$ (power) is the transmit energy in one time unit. The ratio of this transmit energy over a time fraction $\frac{P_{s,k}^{(i)}}{a_{i,k}}$ can be interpreted as the signal-to-noise-plus-interference ratio (the transmit power over the time fraction $a_{i,k}$ is $\frac{P_{s,k}^{(i)}}{a_{i,k}}$ and the noise-plus-interference power is one).

We rewrite problem (10) to its equivalent form as follows:

$$\begin{aligned} &\underset{\{\forall i\in\Phi_N, a_{i,k}, P_{s,k}^{(i)}\}}{\text{maximize}} \sum_{i\in\Phi_N} \log(\sum_{k\in\mathbf{S}_i} f(a_{i,k}, P_{s,k}^{(i)}) - c_i) \\ &\text{s.t.} \quad \text{C1"}: \sum_{k\in\mathbf{S}_i} f(a_{i,k}, P_{s,k}^{(i)}) \geq c_i, \\ &\qquad \text{C2', C3', C4', C5' in (10).} \end{aligned} \tag{13}$$

Theorem 1: Problem (10) and problem (13) are equivalent.

Proof: The transformation from problem (10) to problem (13) is obtained by replacing $a_{i,k}\sum_{s=1}^{M}\log(1+g_{s,k}^{(i)}P_{s,k}^{(i)})$ in (10) with the function f.

As can be seen, for any binary values (i.e., 0 or 1) of $a_{i,k}$ (part of any feasible solutions of either problem (10) or problem (13)), we have $a_{i,k}\sum_{s=1}^{M}\log(1+g_{s,k}^{(i)}P_{s,k}^{(i)}) = f(a_{i,k}, P_{s,k}^{(i)})$. Hence, problems (10) and (13) have identical objective functions and identical feasible regions. In other words, all solutions of problem (10) are also solutions of problem (13) and all solutions of problem (13) are also solutions of problem (10) or the two problems are equivalent. □

A relaxed version of (13) can be written as:

$$\begin{aligned} &\underset{\{\forall i\in\Phi_N, a_{i,k}, P_{s,k}^{(i)}\}}{\text{maximize}} \sum_{i\in\Phi_N} \log(\sum_{k\in\mathbf{S}_i} f(a_{i,k}, P_{s,k}^{(i)}) - c_i) \\ &\text{s.t.} \quad \text{C1", C2', C3', C4' in (13)} \\ &\qquad 0 \leq a_{i,k} \leq 1, \ \ \forall k\in\Psi_K, \forall i\in\Phi_N. \end{aligned} \tag{14}$$

The advantage of (13) over (10) is that its relaxed version (14) is convex w.r.t. $(a_{i,k}, P_{s,k}^{(i)})$. The problem under timesharing (14) also provides an upper bound on the performance of the optimal solution of the NP-hard problem (10).

Theorem 2: Problem (14) is convex.

Proof: See Appendix A. □

Problem (14) itself is practically useful if links can be time-synchronized. In this case, the relaxed variable $a_{i,k}$ can be interpreted as the fraction of time that link i is allowed to use channel f_k [27] [26]. Under the timesharing assumption, problem (14) complies with the Nash bargaining theorem; hence its solution is given by the unique and Pareto-optimal NBS (14) [8].

Theorem 3: If timesharing is allowed and the minimum requested rates are within the network capacity region, a unique NBS exists and is the solution to problem (14).

B. Distributed Optimal Algorithm using Dual Decomposition

In the literature, bargaining games often find applications in centralized resource allocation (e.g., an OFDMA-based single-antenna CRN [28], MIMO-OFDMA broadcast systems [12], channel assignment and power allocation on the downlink of cellular networks [13], etc.). In this section, we develop a distributed algorithm that drives the bargaining process (under timesharing) to the unique and Pareto-optimal NBS.

Because (14) is convex and its Slater's conditions hold [29], strong duality holds, meaning that the solution of the dual problem also solves the primal problem. The Lagrangian of (14) is given in (15), where $\alpha_{i,k}$, γ_i, β_i, and ρ_k are nonnegative Lagrangian multipliers, interpreted as prices for violating the constraints.

The dual problem of (14) is:

$$\text{DP}: \underset{\{\alpha_{i,k},\gamma_i,\beta_i,\rho_k,\forall k\in\mathbf{S}_i,\forall i\in\Psi_N\}}{\text{minimize}} D(\alpha_{i,k},\gamma_i,\beta_i,\rho_k) \tag{17}$$

where D is the dual function, defined as:

$$D = \max_{\{a_{i,k},P_{s,k}^{(i)},\forall k\in\mathbf{S}_i,\forall i\in\Psi_N\}} L(a_{i,k},P_{s,k}^{(i)},\alpha_{i,k},\gamma_i,\beta_i,\rho_k). \tag{18}$$

To facilitate a distributed solution, we decompose the Lagrangian of the primal problem in (7) as in (16), where:

$$\begin{aligned} &L_i(a_{i,k},P_{s,k}^{(i)},\alpha_{i,k},\gamma_i,\beta_i,\rho_k) \\ &\stackrel{\text{def}}{=} \log(\sum_{k\in\Psi_K} a_{i,k}\sum_{s=1}^{M}\log(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}}) - c_i) \\ &+\sum_{k\in\Psi_K}\alpha_{i,k}(-\sum_{s=1}^{M}P_{s,k}^{(i)}+{P_{\text{mask},i}}^{(k)})+\gamma_i(-\sum_{k\in\Psi_K}\sum_{s=1}^{M}P_{s,k}^{(i)}+P_{\max}) \\ &+\beta_i(\sum_{k\in\Psi_K}a_{i,k}\sum_{s=1}^{M}\log(1+\frac{g_{s,k}^{(i)}P_{s,k}^{(i)}}{a_{i,k}})-c_i)-\sum_{k\in\Psi_K}\rho_k a_{i,k}. \end{aligned} \tag{19}$$

To solve (18) for the dual function, each link individually maximizes $L_i(a_{i,k},P_{s,k}^{(i)},\alpha_{i,k},\gamma_i,\beta_i,\rho_k)$ to find the optimal $(a_{i,k}^*, P_{s,k}^{(i)*})$ for given prices $(\alpha_{i,k},\gamma_i,\beta_i,\rho_k)$:

$$\underset{\{a_{i,k}\geq 0, P_{s,k}^{(i)}\geq 0,\forall k\in\Psi_K\}}{\text{maximize}} L_i(a_{i,k},P_{s,k}^{(i)},\alpha_{i,k},\gamma_i,\beta_i,\rho_k). \tag{20}$$

The local problem (20) is convex, and hence can be solved using standard methods like the interior fixed point method. If a central arbitrator is in place (e.g., a base station or spectrum database/broker), after solving the local problem (20), all links report their calculated $(a_{i,k}^*, P_{s,k}^{(i)*})$ to the arbitrator so that the dual function is updated as $L(a_{i,k}^*, P_{s,k}^{(i)*}, \alpha_{i,k},\gamma_i,\beta_i,\rho_k)$.

Because the dual problem DP is convex [14], the arbitrator can solve it efficiently for $(\alpha_{i,k},\gamma_i,\beta_i,\rho_k)$, and then broadcasts these variables. Each link updates its local problem (20) with broadcasted Lagrangian variables. It then solves for $(a_{i,k}^*, P_{s,k}^{(i)*})$ again. The process continues until the dual function converges. This process is illustrated in Fig. 2, and

$$
\begin{aligned}
&L(a_{i,k}, P^{(i)}_{s,k}, \alpha_{i,k}, \gamma_i, \beta_i, \rho_k) = \\
&\sum_{i\in\Phi_N} \log\left(\sum_{k\in\mathbf{S}_i} a_{i,k} \sum_{s=1}^{M} \log(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}}) - c_i\right) + \sum_{i\in\Phi_N}\sum_{k\in\Psi_K} \alpha_{i,k}[-\sum_{s=1}^{M} P^{(i)}_{s,k} + P_{mask}(i,f_k)] \qquad (15)\\
&+\sum_{i\in\Phi_N} \gamma_i[-\sum_{k\in\Psi_K}\sum_{s=1}^{M} P^{(i)}_{s,k} + P_{\max}] + \sum_{i\in\Phi_N} \beta_i[\sum_{k\in\Psi_K} a_{i,k} \sum_{s=1}^{M}\log(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}}) - c_i] + \sum_{k\in\Psi_K} \rho_k(-\sum_{i\in\Phi_N} a_{i,k}+1)\\
&= \sum_{i\in\Phi_N} L_i(a_{i,k}, P^{(i)}_{s,k}, \alpha_{i,k}, \gamma_i, \beta_i, \rho_k) + \sum_{k\in\Psi_K}\rho_k \qquad (16)
\end{aligned}
$$

is referred to as "Arbitrator-Assisted Scheme".

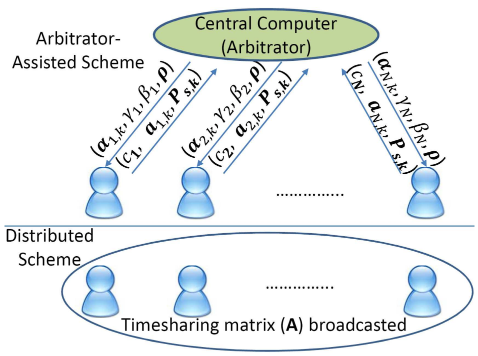

Fig. 2. Arbitrator-assisted and distributed bargaining schemes.

Next, we design a distributed algorithm for problem (14) when no central controller/arbitrator is available. The objective function of the primal problem (14) is continuous and so its dual function is differentiable w.r.t. its Lagrangian variables [29]. Hence, the convex DP problem can be solved with a gradient search algorithm. Specifically, the DP's variables at time $(t+1)$ can be updated as follows:

$$
\begin{aligned}
\alpha_{i,k}^{(t+1)} &= \left[\alpha_{i,k}^{(t)} - \eta\frac{\partial L}{\partial \alpha_{i,k}}\right]^+ = \left[\alpha_{i,k}^{(t)} - \eta(-\sum_{s=1}^{M} P^{(i)(t)*}_{s,k} + P_{\text{mask},i}{}^{(k)})\right]^+\\
\gamma_i{}^{(t+1)} &= \left[\gamma_i{}^{(t)} - \eta\frac{\partial L}{\partial \gamma_i}\right]^+ = \left[\gamma_i{}^{(t)} - \eta(-\sum_{k\in\Psi_K}\sum_{s=1}^{M} P^{(i)(t)*}_{s,k} + P_{\max})\right]^+\\
\beta_i{}^{(t+1)} &= \left[\beta_i{}^{(t)} - \eta\frac{\partial L}{\partial \beta_i}\right]^+\\
&= \left[\beta_i{}^{(t)} - \eta(\sum_{k\in\Psi_K} a^{(t)*}_{i,k}\sum_{s=1}^{M}\log(1+\frac{g^{(i)}_{s,k}P^{(i)(t)*}_{s,k}}{a^{(t)*}_{i,k}}) - c_i)\right]^+\\
\rho_k{}^{(t+1)} &= \left[\rho_k{}^{(t)} - \eta\frac{\partial L}{\partial \rho_k}\right]^+ = \left[\rho_k{}^{(t)} - \eta(-\sum_{i\in\Phi_N} a^{(t)*}_{i,k} + 1)\right]^+
\end{aligned} \qquad (21)
$$

where $\eta > 0$ is a sufficiently small step-size [4] and $(.)^+$ denotes the projection onto the nonnegative orthant.

Observe that the Lagrangian variables $\alpha_{i,k}$, γ_i, and β_i can be calculated and updated using only local information of link i (the fraction of time $a_{i,k}$ that link i tentatively communicates on channel f_k and the power allocated to stream s on channel f_k, $P^{(i)}_{s,k}$). Moreover, the price ρ_k is obtained if other links j broadcast their tentative time fraction $a_{j,k}$ on channel f_k. Our distributed mechanism is shown in Algorithm 1 and illustrated in Fig. 2. The convergence and optimality of Algorithm 1 is claimed as in the next theorem.

Algorithm 1 Distributed Bargaining Algorithm for Computing Optimal Timeshares and Precoders of Link i at Time $(t+1)$:

1: **Input:** $\boldsymbol{a} = (a^{(t+1)}_{1,k}, ..., a^{(t+1)}_{i-1,k}, a^{(t)}_{i+1,k}, ..., a^{(t)}_{N,k}), \forall k \in \Psi_K$
If $t+1 = 0$ (beginning iteration), set $\boldsymbol{a} = (1/N, \dots, 1/N)$
2: **Initialize:** $\tilde{\mathbf{T}}_i^{(t+1)} \leftarrow \tilde{\mathbf{T}}_i^{(t)}$
3: **Computation:**
4: $\forall k \in \Psi_K$, compute transmit and receive precoders ($\mathbf{T}_{i,k}$, $\mathbf{U}^H_{i,k}$), and stream gains $g^{(i)}_{s,k}$ using (9).
5: Update local Lagrangian variables $\alpha^{(t+1)}_{i,k}$, $\gamma_i{}^{(t+1)}$, and $\beta_i{}^{(t+1)}$ using (21).
6: Update price k, $\rho_k^{(t+1)}$ using (21) and timeshares $a^{(t)*}_{j,k}$ from links j, $j \neq i$.
7: Update $L_i(a_{i,k}, P^{(i)}_{s,k}, \alpha^{(t+1)}_{i,k}, \gamma_i{}^{(t+1)}, \beta_i{}^{(t+1)}, \rho_k^{(t+1)})$ (19).
8: Solve problem (20) for $(a^{(t+1)*}_{i,k}, P^{(i)(t+1)*}_{s,k})$.
9: **Broadcast:** tentative timeshares $a^{(t+1)*}_{i,k}$, $\forall k \in \Psi_K$.
10: RETURN $\tilde{\mathbf{T}}^{(t+1)}_{i,k} = \mathbf{T}_{i,k}(\mathbf{P}^{(i)(t+1)*}_{k})^{1/2}, \forall k \in \Psi_K$

Theorem 4: For a sufficiently small $\eta > 0$, Algorithm 1 converges to the globally optimal solution (Pareto-optimal NBS) of problem (14).

Proof: See Appendix B. □

It is worth noting that besides its optimality and distributed implementation, Algorithm 1 greatly reduces the computational time for large networks (large N). Instead of dealing with $N(MK+K)$ variables in the centralized problem (14),

[4] For a gradient-based search, there are various ways to select its step-size to ensure the algorithm's convergence (e.g., constant, diminishing step-size, or using Armijo rule [29]). For instance, Proposition 1.2.3 in [29] states conditions on a constant step-size η to guarantee the search's convergence.

Algorithm 1 involves $MK + K$ variables. In addition to its application in timesharing scenarios, the solution of problem (14) also sheds light on how to derive a distributed solution to the original problem (7), as explained next.

C. Distributed Bargaining Algorithm for Problem (7)

The optimal solution of the relaxed problem provides information on which links wish to access which channels and for what fraction of time. In other words, the preferences of different links over the pool of available channels are revealed. In this section, we exclusively assign a channel to a link by considering preferences of all other links on that channel.

The gradients at the convergence point of Algorithm 1 must be zero if the globally optimal solution to (14) is an interior point of the feasible region. If the solution is a boundary point, the gradient at this point must be positive (negative) along the outwards (inwards) direction of the interior of the feasible region [29]. This fact is conveyed in Equations (22) and (23). For (22) and (23) to be defined at $a_{i,k} = 0$, it must be the case that $P_{s,k}^{(i)} = 0, \forall s = \{1, \ldots, M\}$. Let Δ_i be the amount that the allocated rate for link i (under timesharing) exceeds its demand c_i:

$$\Delta_i \stackrel{\text{def}}{=} \sum_{k \in \Psi_K} \left(a_{i,k} \sum_{s=1}^{M} \log(1 + \frac{g_{s,k}^{(i)} P_{s,k}^{(i)}}{a_{i,k}}) \right) - c_i \qquad (24)$$

When $P_{s,k}^{(i)} > 0$, (22) implies:

$$(\alpha_{i,k} + \gamma_i) \left(\frac{1}{g_{s,k}^{(i)}} + \frac{P_{s,k}^{(i)}}{a_{i,k}}\right) = \frac{1}{\Delta_i} + \beta_i, \forall s = 1, \ldots, M. \qquad (25)$$

As the RHS of (25) does not change for all data streams $1 \leq s \leq M$, link i allocates more power on stream (s, k) with higher gain $g_{s,k}^{(i)}$, and vice versa. This suggests a water-filling-like algorithm for link i to allocate power on channel k. Plugging $1/\Delta_i$ from (25) into (23) and after some manipulations, we get:

$$\frac{\partial L}{\partial a_{i,k}} = \begin{cases} F_{i,k} - \rho_k & \text{if } a_{i,k} > 0 \\ -\rho_k & \text{if } a_{i,k} = 0 \end{cases} \qquad (26)$$

where

$$F_{i,k} \stackrel{\text{def}}{=} \left(\frac{1}{\Delta_i} + \beta_i\right) \sum_{s=1}^{M} \log\left(1 + \frac{g_{s,k}^{(i)} P_{s,k}^{(i)}}{a_{i,k}}\right) - \frac{\alpha_{k,i} + \gamma_i}{a_{i,k}} \sum_{s=1}^{M} P_{s,k}^{(i)}. \qquad (27)$$

Recalling (23), (26) suggests that at the optimal solution, link i should exclusively occupy channel f_k if $F_{i,k} > \rho_k$; otherwise, link i should timeshare the channel with other links or not use f_k if $F_{i,k} < \rho_k$. Note that ρ_k is interpreted as the price of using f_k, which is "flat" for all buyers/links. $F_{i,k}$ can be interpreted as the "payoff" that link i gets from "buying" channel k. If a channel k is to be exclusively allocated to no more than one link, then the link with the highest $F_{i,k}$ should get it. This means the most efficient/needy link (of channel k) wins the channel. Formally, we have the following rule to select the optimal link for f_k:

$$a_{i',k} = \begin{cases} 1 & \text{if } i' = \arg \max_{\forall i \in \Phi_N} F_{i,k} \\ 0 & \text{otherwise} \end{cases} \qquad (28)$$

To execute the above rule in a distributed manner, each link i broadcasts a vector $\boldsymbol{F}_i \stackrel{\text{def}}{=} \{F_{i,1}, \ldots, F_{i,K}\}$. After receiving $\boldsymbol{F}_j$ from every neighbor j, link i can autonomously determine the set of channels $\mathbf{A}_i$ it should select (when comparing $F_{i,k}$ of different links, if a tie happens, we randomly pick any of the links).

Economical Interpretation: Consider $F_{i,k}$ in (27). The first term is the weighted rate that link i can achieve over channel k. The second term is the weighted power that link i invests on channel k. Hence, the "payoff" $F_{i,k}$ is indeed the weighted rate that link i gets from channel k discounted by its allocated (weighted) power. We can observe that the unit price for the discounted power is the total price of violating the PU protection and maximum power budget constraints divided by the timeshare $\frac{\alpha_{k,i} + \gamma_i}{a_{i,k}}$. For the same weighted power and the same scalar $(\frac{1}{\Delta_i} + \beta_i)$, the higher the channel gain $g_{s,k}^{(i)}$ of link i on channel k, the more likely that link i will win the channel. However, if two links have identical gains on channel k and the same weighted power, then the link that has a less amount of extra rate Δ_i (compared with its demand) is likely to win the channel. This fact ensures fair resource allocation. We will see later in Section IV-C that Δ_i does not play any role if we purely maximize network throughput.

After knowing its set of allocated channels $\mathbf{A_i}$, it is necessary for link i to re-solve the power allocation problem to ensure optimality and QoS satisfaction, as follows:

$$\begin{aligned} & \underset{\{P_{s,k}^{(i)} \geq 0, \forall s=1,\ldots,M, \forall k \in \mathbf{A}_i\}}{\text{maximize}} \sum_{k \in \mathbf{A}_i} \sum_{s=1}^{M} \log(1 + g_{s,k}^{(i)} P_{s,k}^{(i)}) \\ & \text{s.t.} \quad \sum_{k \in \mathbf{A}_i} \sum_{s=1}^{M} P_{s,k}^{(i)} \leq P_{\max} \\ & \qquad \sum_{s=1}^{M} P_{s,k}^{(i)} \leq P_{\text{mask},i}^{(k)}, \quad \forall k \in \mathbf{A}_i. \end{aligned} \qquad (29)$$

Problem (29) is convex and hence can be solved efficiently using standard methods. In fact, (29) belongs to the class of generalized water filling problems with multiple water levels (one at each channel), which can be solved efficiently with the algorithms in [30].

If the optimum solution to (29) does not meet the rate demand c_i, link i needs to inform others through a Reallocation Request message (RRM) and increases its bargain to compete for additional channels, i.e., raise its "payoff" vector $\boldsymbol{F}_i$ in (27). Since β_i is the price of violating the minimum rate constraint C3' in (10), it is intuitive to raise β_i by a sufficiently small step-size δ so that i wins only one additional channel l at a time. Algorithmically, δ can be found through a binary search. In our case, we can derive δ and l analytically with no iterations using Algorithm 2.

The idea of Algorithm 2 is to first find the vector of winning "payoffs" ($\boldsymbol{F}_{\max}$) for all channels and then see how far the "payoff" vector $\boldsymbol{F}_i$ of link i is from these values

$$\frac{\partial L}{\partial P^{(i)}_{s,k}} = \frac{g^{(i)}_{s,k}}{\left(\sum\limits_{k\in\Psi_K} a_{i,k}\sum\limits_{s=1}^{M} \log(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}}) - c_i\right)(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}})} - \alpha_{i,k} - \gamma_i + \beta_i\frac{g^{(i)}_{s,k}}{(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}})}\begin{cases} =0 & \text{if } P^{(i)}_{s,k}>0 \\ <0 & \text{if } P^{(i)}_{s,k}=0 \end{cases} \tag{22}$$

$$\frac{\partial L}{\partial a_{i,k}} = \frac{\sum\limits_{s=1}^{M}\left(\log(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}}) - a_{i,k}\frac{\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a^2_{i,k}}}{(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}})}\right)}{\left(\sum\limits_{k\in\Psi_K} a_{i,k}\sum\limits_{s=1}^{M}\log(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}}) - c_i\right)} + \beta_i\sum_{s=1}^{M}\left(\log(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}}) - a_{i,k}\frac{\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a^2_{i,k}}}{(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}})}\right) - \rho_k \begin{cases} =0 & \text{if } 0<a_{i,k}<1 \\ >0 & \text{if } a_{i,k}=1 \\ <0 & \text{if } a_{i,k}=0 \end{cases} \tag{23}$$

Algorithm 2 Finding increment δ for the price of violating the rate demand of link i (problem (7)) and channel l that i is going to acquire:

Input: $\boldsymbol{F}_i, \forall i \in \Phi_N$
Output: δ and l
$\Upsilon_{i,k} \overset{\text{def}}{=} \sum_{s=1}^{M}\log(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}})$
$\boldsymbol{F}_{\max} \overset{\text{def}}{=} \{F_{\max}(1),\ldots,F_{\max}(K)\}$ where $F_{\max}(k) = \max\{F_{i,k}\}, \forall i \in \Phi_N$.
$\boldsymbol{\Theta}_i \overset{\text{def}}{=} \{\Theta_{i,1},\ldots,\Theta_{i,K}\}$ with $\Theta_{i,k} = F_{\max}(k) - F_{i,k}$.
Sort $\boldsymbol{Z}_i \overset{\text{def}}{=} Sort(\boldsymbol{\Theta}_i)$ in ascending order.
Let $Z_i(m)$ is the smallest positive element in $\boldsymbol{Z}_i$.
Set: $\delta = \frac{(Z_i(m)+Z_i(m+1))}{2}$.
Channel that link i is going to acquire is the index of $Z_i(m)$ in $\boldsymbol{\Theta}_i$ before sorting.
RETURN: δ and channel index l.

(vector $\boldsymbol{\Theta}_i$). Recalling Equation (27), if link i wants to win channel k that is currently not allocated for i, then δ must be set to be strictly greater than $\frac{\Theta_{i,k}}{\Upsilon_{i,k}}$. However, link i wants to request only one channel at a time. For that, we sort the elements of $\boldsymbol{\Theta}_i$ in an ascending order, and set δ to the average of the two smallest positive elements.

Using its updated price, $\beta_i = \beta_i + \delta$, link i recalculates the "payoff" vector $\boldsymbol{F}_i$. Consequently, it broadcasts a Reallocation Request message (RRM), containing the index of channel l that i would like to acquire and its updated $\boldsymbol{F}_i$. Upon hearing this message, all links record the new $\boldsymbol{F}_i$. Then, the current "owner" of channel l (say link j) excludes l from its set of allocated channels $\mathbf{A}_j$. Both links i and j re-solve the power allocation problem (29) and check if their demands are met. The process of increasing the bidding price β_i to bargain for additional channels continues until all links get their requested rates.

As aforementioned, we assume that an admission/congestion control mechanism is in place to ensure that the initial channel assignment $\mathbf{A}^0_i$ guarantees the rate demands of all links. In other words, the problem (7) is always feasible. Hence, the above bidding process is guaranteed to stop. If no RRM is heard for a given time duration (set as $Timer$), all links start transmitting on their selected channels. The channel and power allocation for problem (7) is summarized in Algorithm 3.

Algorithm 3 Distributed Bargaining Algorithm to Design Precoders and Allocate Channels for Node i at Time $(t+1)$:

Execute Algorithm 1 (until convergence)
Payoff vector computation $\boldsymbol{F}_i$ (using (27))
Enter channel allocation phase:
Link i broadcasts its payoff vector $\boldsymbol{F}_i$. Then, sets *Timer*
while $Timer$ not expired **do**
  Upon receiving $\boldsymbol{F}_j$ from neighbors j, update the set of allocated channels $\mathbf{A}_i$ using (28).
  Execute the power allocation (29) and check if $R_i \geq c_i$
  if $R_i < c_i$ **then**
    Compute δ and the channel index l
    Set $\beta_i = \beta_i + \delta$ and update $\boldsymbol{F}_i$ using (27) to acquire (additional) channel l
    Broadcast the new $\boldsymbol{F}_i$, RRM and reset *Timer*
  end if
  If a RRM is heard, reset *Timer*
end while
RETURN $\tilde{\mathbf{T}}^{(t+1)}_{i,k} = \mathbf{T}_{i,k}(\mathbf{P}^{(i)(t+1)*}_k)^{1/2}, \forall k \in \mathbf{A}_i$

Remark 3: The network overhead involved in Algorithm 3 is NK scalar values per iteration (K is the size of the payoff vector per link). Specifically, to bargain/bid for channels, each transmitting SU only needs to broadcast its payoff/valuation vector for channels that the SU is competing for. For a reasonable network size, this overhead does not exceed 100 bytes and can easily be amortized over successive data transmissions. For large network, the overhead can be further reduced (at the expense of throughput) by allowing links to skip updating payoff vectors if the change is less than a given threshold. Regarding the algorithm's complexity, each SU needs to solve a convex problem using any conventional solver in polynomial time. We assume secondary users are truthful and cooperative when broadcasting their "payoffs". Dealing with untruthful users is out of the scope of this work.

IV. Network Throughput Maximization

In this section, we develop a distributed algorithm for NET-MAX, as defined in Section II-C. First, we use the zero-duality gap [5] [6] [7] to derive a centralized solution, which can serve as a performance benchmark.

$$L(a_{i,k}, P^{(i)}_{s,k}, \alpha_{i,k}, \gamma_i, \beta_i, \rho_k) = \sum_{k\in\Psi_K}\sum_{i\in\Phi_N} a_{i,k}\sum_{s=1}^{M}\log(1+g^{(i)}_{s,k}P^{(i)}_{s,k}) + \sum_{k\in\Psi_K}\sum_{i\in\Phi_N}\alpha_{i,k}[-\sum_{s=1}^{M}P^{(i)}_{s,k}+P^{(k)}_{\text{mask},i}]$$
$$+\sum_{k\in\Psi_K}\sum_{i\in\Phi_N}\gamma_i[\frac{P_{\max}}{K}-\sum_{s=1}^{M}P^{(i)}_{s,k}] + \sum_{k\in\Psi_K}\sum_{i\in\Phi_N}\beta_i[a_{i,k}\sum_{s=1}^{M}\log(1+g^{(i)}_{s,k}P^{(i)}_{s,k})-\frac{c_i}{K}] + \sum_{k\in\Psi_K}\rho_k(-\sum_{i\in\Phi_N}a_{i,k}+1) \quad (30)$$
$$= \sum_{k\in\Psi_K} L_k(a_{i,k}, P^{(i)}_{s,k}, \alpha_{i,k}, \gamma_i, \beta_i, \rho_k) \quad (31)$$

A. Centralized Algorithm with Zero-duality Gap

The Lagrangian function of the NET-MAX problem (11) is written in (30), and is decomposed w.r.t. channels as in (31) with:

$$L_k(a_{i,k}, P^{(i)}_{s,k}, \alpha_{i,k}, \gamma_i, \beta_i, \rho_k)$$
$$\stackrel{\text{def}}{=} \sum_{i\in\Phi_N} a_{i,k}\sum_{s=1}^{M}\log(1+g^{(i)}_{s,k}P^{(i)}_{s,k})$$
$$+\sum_{i\in\Phi_N}\alpha_{i,k}[-\sum_{s=1}^{M}P^{(i)}_{s,k}+P^{(k)}_{\text{mask},i}] + \sum_{i\in\Phi_N}\gamma_i[\frac{P_{\max}}{K}-\sum_{s=1}^{M}P^{(i)}_{s,k}]$$
$$+\sum_{i\in\Phi_N}\beta_i[a_{i,k}\sum_{s=1}^{M}\log(1+g^{(i)}_{s,k}P^{(i)}_{s,k})-\frac{c_i}{K}] + \rho_k(-\sum_{i\in\Phi_N}a_{i,k}+1). \quad (32)$$

As the duality gap vanishes for a large number of channels, one can solve (11) by solving K subproblems for maximizing $L_k(a_{i,k}, P^{(i)}_{s,k}, \alpha_{i,k}, \gamma_i, \beta_i, \rho_k)$. Since a channel k cannot be allocated to more than one link, it is allocated to a link i that maximizes $L_k(a_{i,k}, P^{(i)}_{s,k}, \alpha_{i,k}, \gamma_i, \beta_i, \rho_k)$ among all links [7].

B. Dual Decomposition with Timesharing

Following a similar procedure to the one used to convert problem (7) to (13) in Section III-A, the problem below is equivalent to the original NET-MAX problem (11):

$$\underset{\{a_{i,k}, P^{(i)}_{s,k}\}}{\text{maximize}} \sum_{i\in\Phi_N}\sum_{k\in\mathbf{S}_i} a_{i,k}\sum_{s=1}^{M}\log(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}}) \quad (33)$$
$$\text{s.t.} \quad \text{C1', C2', C3', C4', C5' in (10).}$$

By relaxing the variables $a_{i,k}$'s, we end up with the following formulation for the optimal timesharing version of NET-MAX:

$$\underset{\{a_{i,k}, P^{(i)}_{s,k}\}}{\text{maximize}} \sum_{i\in\Phi_N}\sum_{k\in\mathbf{S}_i} a_{i,k}\sum_{s=1}^{M}\log(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}})$$
$$\text{s.t.} \quad \text{C1', C2', C3', C4'in (10)} \quad (34)$$
$$0 \le a_{i,k} \le 1, \quad \forall k\in\mathbf{S}_i, \forall i\in\Phi_N.$$

Beside its practical application, (34) also provides an upper bound on the performance of the optimal solution of the original NET-MAX.

Theorem 5: Problem (34) is convex.

The proof of the above theorem is similar to that of Theorem 2, and is omitted for brevity. Using a similar dual decomposition as in Section III-B, we derive a distributed algorithm that attains the globally optimal solution of (34). Specifically, the transmitter of link i solves the following local problem (for simplicity, we use the same notation as in Section III):

$$\underset{\{a_{i,k}\ge 0, P^{(i)}_{s,k}\ge 0, \forall k\in\mathbf{S}_i\}}{\text{maximize}} D_i(a_{i,k}, P^{(i)}_{s,k}, \alpha_{i,k}, \gamma_i, \beta_i, \rho_k) \quad (35)$$

where

$$D_i(a_{i,k}, P^{(i)}_{s,k}, \alpha_{i,k}, \gamma_i, \beta_i, \rho_k) = \sum_{k\in\Psi_K} a_{i,k}\sum_{s=1}^{M}\log(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}})$$
$$+\sum_{k\in\Psi_K}\alpha_{i,k}(-\sum_{s=1}^{M}P^{(i)}_{s,k}+P^{(k)}_{\text{mask},i}) + \gamma_i(-\sum_{k\in\Psi_K}\sum_{s=1}^{M}P^{(i)}_{s,k}+P_{\max})$$
$$+\beta_i(\sum_{k\in\Psi_K} a_{i,k}\sum_{s=1}^{M}\log(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}})-c_i) - \sum_{k\in\Psi_K}\rho_k a_{i,k}. \quad (36)$$

The Lagrangian multipliers (prices) are updated using the same rule in (21) (since (34) and (14) have identical constraints).

Theorem 6: If each link solves problem (35), updates Lagrangian multipliers using (21), and broadcasts its tentative timeshare, then the network converges to the optimal solution of the NET-MAX (34) under timesharing.

Proof: Similar to the proof of Theorem 4. □

C. Distributed Algorithm for NET-MAX

Note that if one decomposes (8) w.r.t. various links in its dual domain to obtain a distributed algorithm, the resulting subproblems in the dual domain still face the same combinatorial selection of channels as in the primal problem. Instead, we develop a distributed (suboptimal) algorithm for the problem (8) using the optimal solution of the above relaxed problem.

Using the arguments in Section III-C, one can compute a "payoff" vector for NET-MAX, denoted by $\boldsymbol{Y}_{i,k}$, which is analogous to the role of $\boldsymbol{F}_{i,k}$ for BF-CMIMO:

$$Y_{i,k} \stackrel{\text{def}}{=} (1+\beta_i)\sum_{s=1}^{M}\log\left(1+\frac{g^{(i)}_{s,k}P^{(i)}_{s,k}}{a_{i,k}}\right) - \frac{\alpha_{i,k}+\gamma_i}{a_{i,k}}\sum_{s=1}^{M}P^{(i)}_{s,k}. \quad (37)$$

Channels are then exclusively allocated as follows:

$$a_{i',k} = \begin{cases} 1 & \text{if } i' = \arg \max_{\forall i \in \Phi_N} \; Y_{i,k} \\ 0 & \text{otherwise} \end{cases} \tag{38}$$

The economical interpretation of the above rule resembles that of BF-CMIMO. However, the weight factor of the achievable rate $(1 + \beta_i)$ under NET-MAX differs from that of BF-CMIMO $(1/\Delta_i + \beta_i)$. When maximizing network throughput and ignoring fairness aspects, the amount of extra rate Δ_i does not play any role in channel bargaining/assignment. Under NET-MAX, regardless of how much rate link i gets, it wins a given channel if it is the most efficient user of that channel.

Now, links follow the same procedure of re-solving the power allocation problem (29), checking if all rate demands are met, and sending RRM if necessary, as in Algorithm 3.

V. Numerical Results

We simulated a CMIMO network in which nodes are randomly distributed on a square field of length 100 m. The following simulation parameters are in accordance with FCC specifications for Wireless LANS (FCC Section 15.247 [31]). The free-space attenuation factor is 2 and channel bandwidth is 16 MHz. We set $P_{\max} = 1$ W and $P_{\text{mask}}(f_k) = 0.5$ W $\forall f_k$. Noise floor (around -174 dBm) plus PUs interference is -100 dBm/Hz. Without loss of generality, we set the rate demands of all links to 2 bits/s/Hz. The spreading angles of arriving signals vary from $-\pi/5$ to $\pi/5$, assuming a rich-scattering environment for MIMO (multiplexing gain) operation. All plots are obtained from taking the average of 20 simulations runs. In each run, the node location, channel matrices are randomly regenerated.

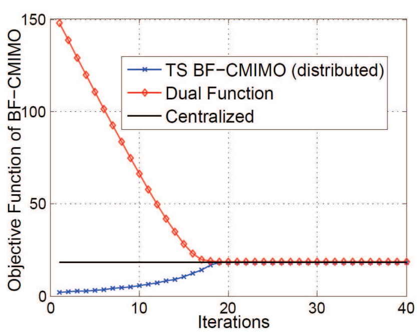

Fig. 3. Convergence of BF-CMIMO under timesharing (TS).

To evaluate the optimality and convergence of the distributed algorithms under timesharing (TS), we consider a network of 10 links, 15 channels, and 4 antennas per node. To capture spectrum heterogeneity, we assume channels i, $(i+1)$, $(i+2)$ are not available to link i. Figures 3 and 4 depict the dual functions and network throughput vs. iterations of the two distributed algorithms for BF-CMIMO and NET-MAX under TS, respectively. The distributed algorithm of TS BF-CMIMO converges to the optimal centralized solution after 18 iterations. Similarly, the distributed algorithm of TS NET-MAX converges to the optimal network throughput under the centralized solution after 27 iterations. Using the

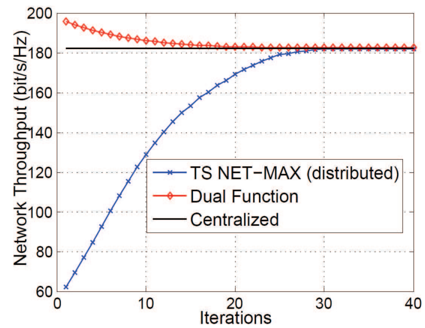

Fig. 4. Convergence of NET-MAX under timesharing (TS).

approach in [32], one can quantify the convergence speed of the two distributed algorithms. Because of its logarithmic objective function, the distributed algorithm of BF-CMIMO converges faster than that of NET-MAX. Under exclusive channel allocation (no TS), we observed that the heuristic algorithms for BF-CMIMO and NET-MAX often need less than 3 additional iterations to reallocate channels.

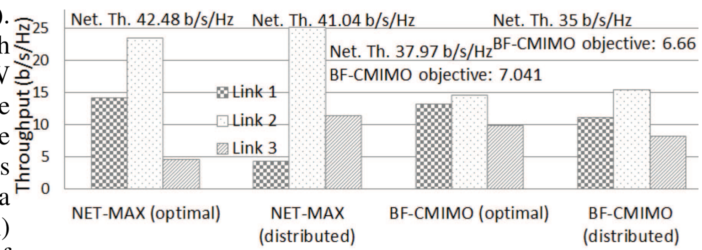

Fig. 5. Distributed BF-CMIMO and NET-MAX algorithms vs. optimal solutions (via exhaustive search).

To compare the performance of the heuristic algorithms for BF-CMIMO and NET-MAX with their optimal solutions under the exclusive channel occupancy policy, we run an exhaustive search on a small network of 3 links, 8 channels, and 2 antennas per node. Note that due to the excessive computational complexity prohibits of exhaustive search, we are not able to plot the performance of the exhaustive search vs. the number of antennas/users. However, the performance of the proposed approach (with much less complexity) have been plotted vs. the number of antennas/users in Figures 8, 9, 10, 11, 12, 13. Note that, in these figures, the performance of the proposed approach is compared with that of the timesharing algorithms which offer upper-bounds for those of the exhaustive search.

Fig. 5 shows that the objective function of BF-CMIMO under the proposed algorithm is 6.66, compared with 7.04 for the optimal solution. Similarly, the total throughput under the NET-MAX distributed algorithm is 41.04 bits/s/Hz, compared with 42.48 bits/s/Hz for the optimal solution of NET-MAX. The proposed algorithm achieves 95% of the optimal values.

Figures 6 and 7 compare the network throughput of NET-MAX under our distributed algorithm, the zero-duality (centralized) based algorithm (subsection IV-A), and an exhaustive search solution. For a small number of channels

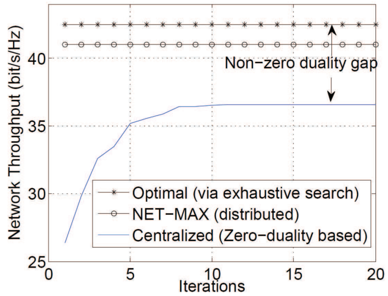

Fig. 6. Non-zero duality gap in NET-MAX for a small number of channels (K=8).

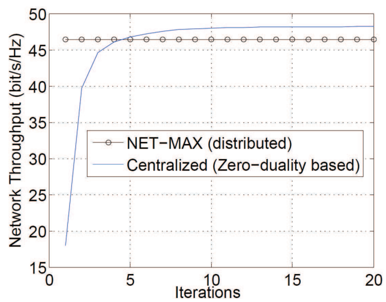

Fig. 7. Zero-duality based (centralized) algorithm with large number of channels (K=15) vs. (proposed) distributed NET-MAX.

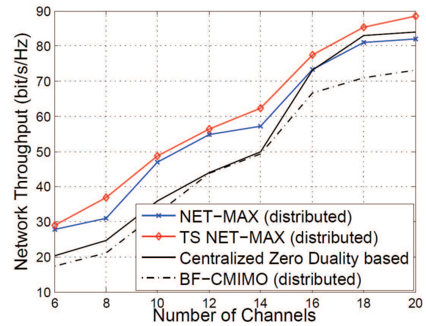

Fig. 8. Throughput vs. number of channels.

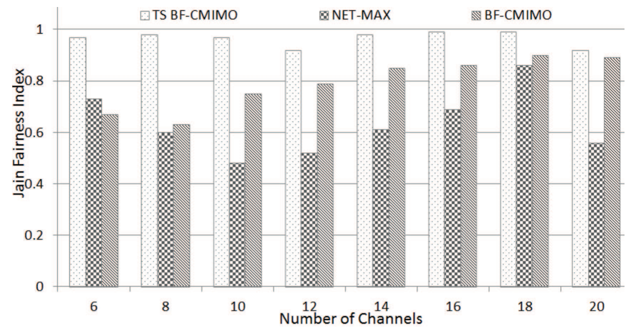

Fig. 9. Jain's fairness index vs. number of channels.

($K = 8$), even in a centralized manner, the zero-duality-based algorithm does not converge to the optimal solution, obtained via exhaustive search (Figure 6). However, the proposed distributed algorithm achieves higher throughput than the zero-duality-based and its performance is close to that of the optimal solution. When K is large, e.g., $K = 15$, the zero-duality-based algorithm achieves higher throughput than the proposed distributed algorithm. This suggests that the zero-duality-based one may converge to the optimal solution (thanks to zero-duality gap). However, the gap between distributed algorithm and the the zero-duality-based one is marginal.

Figures 8 and 9 respectively show the network throughput and Jain's fairness index[5] vs. K a network of 5 links and 2 antennas per node. As can be seen, for a small K, the distributed NET-MAX algorithm outperforms the centralized zero-duality based algorithm in terms of network throughput.

Figure 10 depicts the total network throughput vs. the number of links under BF-CMIMO and NET-MAX, with or without TS ($K = 15$, $M = 2$). For all algorithms, the throughput increases with the number of links (N). This is partially due to the higher link and frequency diversity gains. By directly maximizing the network throughput, the TS NET-MAX distributed algorithm achieves the highest throughput. The TS BF-CMIMO distributed algorithm is the second best in terms of throughout. It achieves about 87% of the TS NET-MAX throughput, in these examples. When channels are exclusively allocated, the distributed algorithm of NET-MAX achieves 78% of its upper bound obtained by TS NET-MAX. The distributed algorithm of BF-CMIMO when timesharing is not allowed achieves 80% of the throughout that can be obtained under NET-MAX.

Figure 11 depicts Jain's fairness index of the four distributed algorithms (BF-CMIMO and NET-MAX with and without TS) vs. the number of links with $K = 15$ and $M = 2$. Algorithms that rely on NB (with or without TS) achieve significantly better fairness than those of NET-MAX. As N increases, fairness under NET-MAX (with or without TS) decreases. However, BF-CMIMO algorithms maintain quite stable fairness for different network sizes. This is because under BF-CMIMO, channels (or their timeshares) are allocated while accounting for the amount of extra rate Δ_i. Jain's index for the distributed algorithm under BF-CMIMO with exclusive channel allocation achieves 81% of its upper bound obtained under TS.

Figures 12 and 13 depict the network throughput and Jain's fairness index vs. the number of antennas per node for a network of 5 links and 15 channels. As expected, in Fig. 12, the network throughput under the three algorithms increases w.r.t. the number of antennas per node. However, in Fig. 13, with a larger antenna array, the NET-MAX achieves slightly lower Jain's fairness index. This is because with a higher number of antenna per node, the throughput per link increases and that exacerbates the unfairness among links.

VI. Conclusions

In this paper, we developed distributed algorithms to jointly allocate channels (under the exclusive channel occu-

[5]Jain's fairness index of N links is a fairness indicator, defined as $\mathcal{J}(t_1, t_2, \ldots, t_N) = \frac{(\sum_{i=1}^{N} t_i)^2}{N \cdot \sum_{i=1}^{N} {t_i}^2}$, where t_i is the throughput for link i.

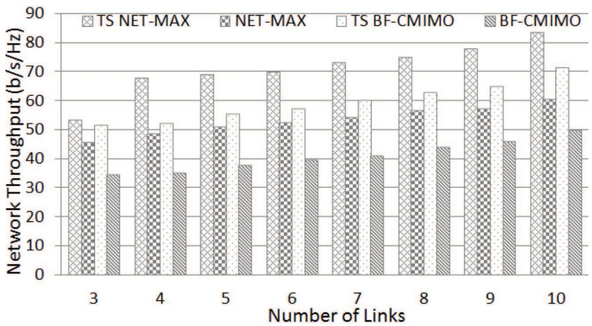

Fig. 10. Network throughput under BF-CMIMO and NET-MAX, with and without TS.

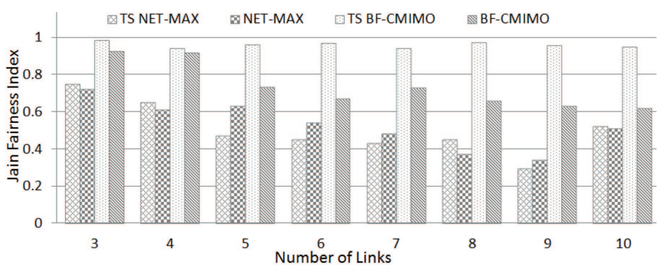

Fig. 11. Jain's fairness index under BF-CMIMO and NET-MAX, with and without TS.

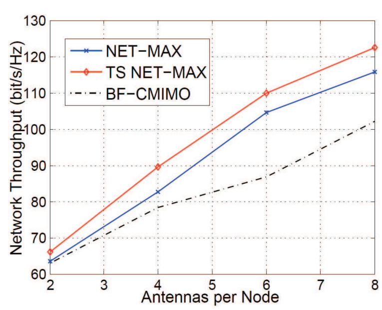

Fig. 12. Throughput vs. number of antennas.

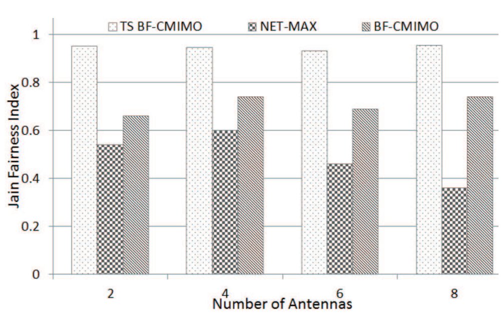

Fig. 13. Jain's fairness index vs. number of antennas.

pancy), and optimize power allocation and antenna patterns (through precoding matrices) for cognitive MIMO networks. The proposed algorithms allow cognitive MIMO links to propose their rate demands, cooperate and bargain to get their channel assignment, and optimize their precoders to either maximize fairness or network throughput. Under timesharing, the distributed algorithms were proved to converge to their network-wide globally optimal solutions. The algorithms under timesharing revealed preferences of different links on a channel that guide heuristic algorithms to allocate channels under the exclusive-channel occupancy policy. Simulations indicated that these heuristic algorithms perform very close to their optimal solutions. The proposed bargaining framework significantly improves user fairness with moderate throughput reduction. We believe this work provides a theoretical foundation for integrating MIMO spatial multiplexing capability into the next-generation DSA networks. Our future work will focus on designing protocols that incentivize links to broadcast their true timeshares.

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APPENDIX A
PROOF OF THEOREM 2

First, using L'Hospital's rule, one can verify that $f(a_{i,k}, P_{s,k}^{(i)})$ is continuous w.r.t. $a_{i,k}$ at $a_{i,k} = 0$. Next, let $v(a_{i,k}, P_{s,k}^{(i)}) \stackrel{\text{def}}{=} \sum_{k \in \mathbf{S}_i} f(a_{i,k}, P_{s,k}^{(i)})$ then $v(a_{i,k}, P_{s,k}^{(i)})$ is a concave function w.r.t. $(a_{i,k}, P_{s,k}^{(i)})$, as it is the summation of perspective functions [14] of the logarithmic concave functions $\log(1+x)$ (the perspective function of a concave function is concave [14]). Hence, constraint C1" defines a convex region. Consequently, the feasible region of problem (14) is an intersection of half-spaces and convex regions, and is hence convex w.r.t. $(a_{i,k}, P_{s,k}^{(i)})$.

The objective function is the summation of N functions $g(a_{i,k}, P_{s,k}^{(i)})$ where:

$$g(a_{i,k}, P_{s,k}^{(i)}) \stackrel{\text{def}}{=} \log(\sum_{k \in \mathbf{S}_i} f(a_{i,k}, P_{s,k}^{(i)}) - \mathrm{c}_i).$$

We will show that $g(a_{i,k}, P_{s,k}^{(i)})$ is concave w.r.t. $(a_{i,k}, P_{s,k}^{(i)})$. Let's rewrite g as a composite function $g = h(v(a_{i,k}, P_{s,k}^{(i)}))$ where $h(x) \stackrel{\text{def}}{=} \log(x - c_i)$. We know that $v(a_{i,k}, P_{s,k}^{(i)})$ is a concave function w.r.t. $(a_{i,k}, P_{s,k}^{(i)})$. Additionally, h is a nondecreasing function. Hence, the composite function $g = h(v(a_{i,k}, P_{s,k}^{(i)}))$ is concave [14]. □

APPENDIX B
PROOF OF THEOREM 4

Algorithm 1 is a form of the solution to the Network Utility Maximization (NUM) problem in [33] and [34]. Following the same procedure in [33] and [34] to establish the convergence of Algorithm 1 can be very cumbersome since our utility function has a much more complicated form than those in [33], [34]. Here, we present an alternative proof. Specifically, we need to show:

1) The distributed algorithm converges.
2) The converged point is the globally optimal solution of the problem (14).

For the convergence, we prove that the dual function $D(\alpha_{i,k}, \gamma_i, \beta_i, \rho_k)$ is nonincreasing and bounded from below. It is clear that the dual function is always lower-bounded by the objective function of the primal problem which again can be bounded from below by its value at any feasible solution. Next, consider the difference of the dual function between two consecutive iterations $(\alpha_{i,k}^{(t)}, \gamma_i^{(t)}, \beta_i^{(t)}, \rho_k^{(t)})$ and $(\alpha_{i,k}^{(t+1)}, \gamma_i^{(t+1)}, \beta_i^{(t+1)}, \rho_k^{(t+1)})$:

$$\begin{aligned} &D(\alpha_{i,k}^{(t+1)}, \gamma_i^{(t+1)}, \beta_i^{(t+1)}, \rho_k^{(t+1)}) - D(\alpha_{i,k}^{(t)}, \gamma_i^{(t)}, \beta_i^{(t)}, \rho_k^{(t)}) \\ &= L(a_{i,k}^{(t+1)}, P_{s,k}^{(i)(t+1)}, \alpha_{i,k}^{(t+1)}, \gamma_i^{(t+1)}, \beta_i^{(t+1)}, \rho_k^{(t+1)}) \\ &- L(a_{i,k}^{(t)}, P_{s,k}^{(i)(t)}, \alpha_{i,k}^{(t)}, \gamma_i^{(t)}, \beta_i^{(t)}, \rho_k^{(t)}) \end{aligned} \tag{39}$$

$$\begin{aligned} &= L(a_{i,k}^{(t+1)}, P_{s,k}^{(i)(t+1)}, \alpha_{i,k}^{(t+1)}, \gamma_i^{(t+1)}, \beta_i^{(t+1)}, \rho_k^{(t+1)}) \\ &\quad - L(a_{i,k}^{(t+1)}, P_{s,k}^{(i)(t+1)}, \alpha_{i,k}^{(t)}, \gamma_i^{(t)}, \beta_i^{(t)}, \rho_k^{(t)}) \\ &\quad + L(a_{i,k}^{(t+1)}, P_{s,k}^{(i)(t+1)}, \alpha_{i,k}^{(t+1)}, \gamma_i^{(t)}, \beta_i^{(t)}, \rho_k^{(t)}) \\ &\quad - L(a_{i,k}^{(t)}, P_{s,k}^{(i)(t)}, \alpha_{i,k}^{(t)}, \gamma_i^{(t)}, \beta_i^{(t)}, \rho_k^{(t)}) \\ &\le -\eta[\frac{\partial L}{\partial \alpha_{1,1}}, \ldots, \frac{\partial L}{\partial \alpha_{N,K}}, \frac{\partial L}{\partial \beta_1}, \ldots, \frac{\partial L}{\partial \beta_N}, \ldots, \frac{\partial L}{\partial \rho_1}, \ldots, \frac{\partial L}{\partial \rho_K}]^T \\ &\quad \times [\frac{\partial L}{\partial \alpha_{1,1}}, \ldots, \frac{\partial L}{\partial \alpha_{N,K}}, \frac{\partial L}{\partial \beta_1}, \ldots, \frac{\partial L}{\partial \beta_N}, \ldots, \frac{\partial L}{\partial \rho_1}, \ldots, \frac{\partial L}{\partial \rho_K}] + 0 \\ &\le 0 \end{aligned} \tag{40}$$

The inequality follows from two facts. First, L is convex w.r.t. $(\alpha_{i,k}, \gamma_i, \beta_i, \rho_k)$ and for sufficiently small stepsize η, the descent direction update in (21) always reduces L while fixing $(a_{i,k}, P_{s,k}^{(i)})$. Second, $(a_{i,k}^{(t)}, P_{s,k}^{(i)(t)})$ are maximizers of $L(a_{i,k}^{(t)}, P_{s,k}^{(i)(t)}, \alpha_{i,k}^{(t)}, \gamma_i^{(t)}, \beta_i^{(t)}, \rho_k^{(t)})$ (found from solving N parallel local problems (20) while fixing $(\alpha_{i,k}^{(t)}, \gamma_i^{(t)}, \beta_i^{(t)}, \rho_k^{(t)})$).

Next, we show that the converged point meets the K.K.T. conditions of the convex problem (14), hence it is the globally optimal solution. From inequality (40), at the converged point, the equality takes place. Thus,

$$\begin{aligned} &L(a_{i,k}^{(t+1)}, P_{s,k}^{(i)(t+1)}, \alpha_{i,k}^{(t+1)}, \gamma_i^{(t+1)}, \beta_i^{(t+1)}, \rho_k^{(t+1)}) \\ &= L(a_{i,k}^{(t+1)}, P_{s,k}^{(i)(t+1)}, \alpha_{i,k}^{(t)}, \gamma_i^{(t)}, \beta_i^{(t)}, \rho_k^{(t)}) \end{aligned} \tag{41}$$

and

$$\begin{aligned} &L(a_{i,k}^{(t+1)}, P_{s,k}^{(i)(t+1)}, \alpha_{i,k}^{(t+1)}, \gamma_i^{(t)}, \beta_i^{(t)}, \rho_k^{(t)}) \\ &= L(a_{i,k}^{(t)}, P_{s,k}^{(i)(t)}, \alpha_{i,k}^{(t)}, \gamma_i^{(t)}, \beta_i^{(t)}, \rho_k^{(t)}). \end{aligned} \tag{42}$$

Since L is concave w.r.t. $(a_{i,k}, P_{s,k}^{(i)})$ (while fixing $(\alpha_{i,k}, \gamma_i, \beta_i, \rho_k)$) and convex w.r.t. $(\alpha_{i,k}, \gamma_i, \beta_i, \rho_k)$ (while fixing $(a_{i,k}, P_{s,k}^{(i)})$), (41) and (42) happen if and only if the gradient of $L(a_{i,k}^{(t+1)}, P_{s,k}^{(i)(t+1)}, \alpha_{i,k}, \gamma_i, \beta_i, \rho_k)$ at $(\alpha_{i,k}^{(t+1)}, \gamma_i^{(t+1)}, \beta_i^{(t+1)}, \rho_k^{(t+1)})$ vanishes and $(a_{i,k}^{(t+1)}, P_{s,k}^{(i)(t+1)})$ are also maximizers of $L(a_{i,k}, P_{s,k}^{(i)}, \alpha_{i,k}^{(t)}, \gamma_i^{(t)}, \beta_i^{(t)}, \rho_k^{(t)})$. In other words:

$$\begin{aligned} &\frac{\partial L}{\partial \alpha_{i,k}} = \frac{\partial L}{\partial \beta_i} = \frac{\partial L}{\partial \rho_k} = \frac{\partial L}{\partial a_{i,k}} = \frac{\partial L}{\partial P_{s,k}^{(i)}} = 0 \\ &\forall i \in \Phi_N, \forall k \in \Psi_K, \text{ and } s = \{1, \ldots, M\}. \end{aligned} \tag{43}$$

This is exactly the K.K.T. conditions of the convex problem (14). □

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